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Multi-Attribute Decision-making Based on Pythagorean Fuzzy Numbers and its Application in Hotel Evaluations

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ABSTRACT

This article describes a method for assessing hotel management using Pythagorean Fuzzy Numbers inside a Multi-attribute Decision-Making framework. It offers a grey relational analysis projection to deal with scenarios in which attribute values fluctuate within the Pythagorean Fuzzy Set range and attribute weights are unknown. The study developed numerous operational rules and computed the anticipated value and the Hamming distance between two Pythagorean Fuzzy Sets. The information entropy method was then utilized to calculate attribute weights, creating the Grey Relational Analysis and Grey Relational Projection methodologies. Alternatives were rated based on their proximity to the Positive Ideal Target using Grey Relational Projection values from both positive and negative ideal solutions for each alternative. The validity of this model was verified through a case study on hotel management evaluation, demonstrating its practicality and effectiveness, and Comparative Analysis involving the adjustment of criteria weight coefficients.

1. Introduction

The worldwide landscape has become more digitalized and networked, leading to a substantial increase in the complexity of decision-making processes in various industries, including technology, healthcare, and finance. Traditional decision-making methods frequently need to improve when faced with large volumes and ambiguity of modern data streams. Pythagorean Fuzzy Numbers (PFNs) offer an enhanced solution to uncertainty and partial truth, making them an effective tool for navigating these complications. PFNs' flexibility and robustness enable better modeling of uncertain information, making them especially useful when standard methods cannot fully capture the specifics of real-world data. Building upon the foundational concepts of fuzzy logic, Pythagorean Fuzzy Numbers (PFNs) present an advanced framework that extends these principles

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by offering a more flexible and nuanced way to represent uncertainty. Unlike traditional fuzzy numbers constrained by the linear condition of $u + v \ll 1$(where u and v represent the degrees of membership and non-membership, respectively), PFNs employ a quadratic constraint $u^2 + v^2 \ll 1$. This modification allows for a richer and more detailed characterization of uncertain and imprecise information, which is particularly beneficial in MCDM applications. By integrating PFNs, we can enhance the analytical capabilities of decision-making models, especially in environments where clarity and certainty are limited, thus facilitating more informed and practical decision processes.

2. Literature review

Multi-criteria decision-making (MCDM) is a combinational scheme that allows decision-makers to determine appropriate alternatives based on various criteria. Zadeh [1] initiated the fuzzy theory in 1965 to overcome uncertainties and fuzziness. In 1986, Atanassov [2] expanded the idea of the fuzzy theory to postulate an Intuitionistic Fuzzy Set (IFS), namely, u ϵ [0,1] and v ϵ [0,1]. The IFS guarantees that the sum of the membership point (u) and the non-membership point (v) will be ≤ 1. This Intuitionistic Fuzzy Set (IFS) is used extensively in various fields, including Multi-attribute Decision-making (MADM). However, the sum of u and v will be ≥ 1 if DM expresses support for u in x as 0.9 and in opposition to u in x as 0.6. As a result, the paired numbers (0.9, 0.6) cannot be used in an IFS. To compensate for this deficiency and meet the requirement, Yager designed a Pythagorean Fuzzy Set (PFS) element, which has two functions: u and v (u2 + v2 ≤1). The main difference between a PFS and Atanassov's IFS is that the former must satisfy the situation in which the square sum of the u degree and the v degree is ≤ 1, while the sum of the two degrees must not be ≤ 1, whereas the latter must satisfy the situation in which the sum of the two degrees is ≤ 1. Following this finding, Zhang and Xu [3] reported on the PFS TOPSIS for MADM. Shannon [4] explained the GRA, and Zhang and Xu [3] extended the PFS to interval-valued PFSs (IVPFSs). Yager [5] utilized PFSs to solve the recommender system. Huang et al. [6] explained the entropy weight calculation, and Yager [7] utilized the PFS and conducted a case study of Netflix movie data. In addition, Zhang explained the entropy weight [8] for PFS. In addition, different discussions [9, 10, 11] have been made on the multi-criteria decision. Deng [12] elaborates on the Grey Relational Analysis. Chen et al. [13] utilized the TOPSIS method and similarity measures between IFS. Chatterjee and Kara [14] presented an integrated CRITIC and GRA approach for investment decision-making, demonstrating its efficacy in evaluating multiple investment options based on various criteria. Jokić et al. [15] utilized a combination of the LBWA and fuzzy MABAC models to optimize the selection of fire positions for mortar units. Radovanović et al. [16] introduced a hybrid model, LMAW-G-EDAS, to aid in selecting assault rifles for military use. In agriculture, Nedeljković et al. [17] employed a multicriteria approach to enhance the establishment of fruit orchards, explicitly focusing on plum variety selection. Imran et al. [18] proposed a multi-criteria group decision-making approach using interval-valued intuitionistic fuzzy information combined with Aczel-Alsina Bonferroni means for robot selection. Bouraima et al. [19] applied the AROMAN MCDM approach to devise sustainable healthcare system devolution strategies. Yazdi and Komasi [20] explored the performance of COVID-19 management in the Americas using artificial intelligence integrated with MCDM. Badi et al. [21] investigated vendor-managed inventory optimization in multi-tier distribution systems using an MCDM approach. Nzotcha and Kenfack [22] comprehensively assess renewable energy investment strategies utilizing Interval Type-2 (IT2) fuzzy DEMATEL, TOPSIS, and grey relational analysis. In the realm of photovoltaic solar farm site selection, Noorollahi et al. [23] employ a GIS-based approach combined with fuzzy Boolean logic and grey relational analysis. Zhong et al. [24] introduce a hybrid fuzzy multi-criteria decision analysis (MCDA) approach, which effectively evaluates the performance of park-level integrated energy systems. Moreover, the potential of offshore wind energy is explored through spatial MCDA by Vinhoza and Schaeffer [25], who utilize the Analytic Hierarchy Process (AHP) and grey relational analysis to assess wind energy potential. The theoretical underpinnings of MCDM methods are well-documented by Taherdoost and Madanchian [26], who provide a detailed overview of various MCDM concepts and methodologies. Additionally, Cai et al. [27] offer a survey on collaborative decision-making, highlighting its growing relevance in engineering applications. Lastly, Liao et al. [28] explore interval analysis techniques and their fuzzy extensions within MCDM. Their overview presents a detailed examination of how fuzzy systems can enhance decision-making accuracy in situations characterized by uncertainty and imprecision.

The outline of this paper is as follows: Part 1 reviews some basic definitions. Part 2 introduces PFS and explains some arithmetical operations. Part 3 presents a novel strategy for solving MADM issues with PFS. Part 4 gives a practical explanation of a numerical problem. Finally, Part 5 presents the concluding remarks.

2.1 Materials and Methods

Given the complexity of the research problem, a model incorporating PFS and Grey Relational Analysis (GRA) was developed to aid in the selection of hotels. Initially, experts defined the analysis criteria. The GRA method was then utilized to determine the weight coefficients of these criteria. The GRA method was applied to evaluate and select the most suitable hotel alternatives. See the flowchart of the study in Figure 1.

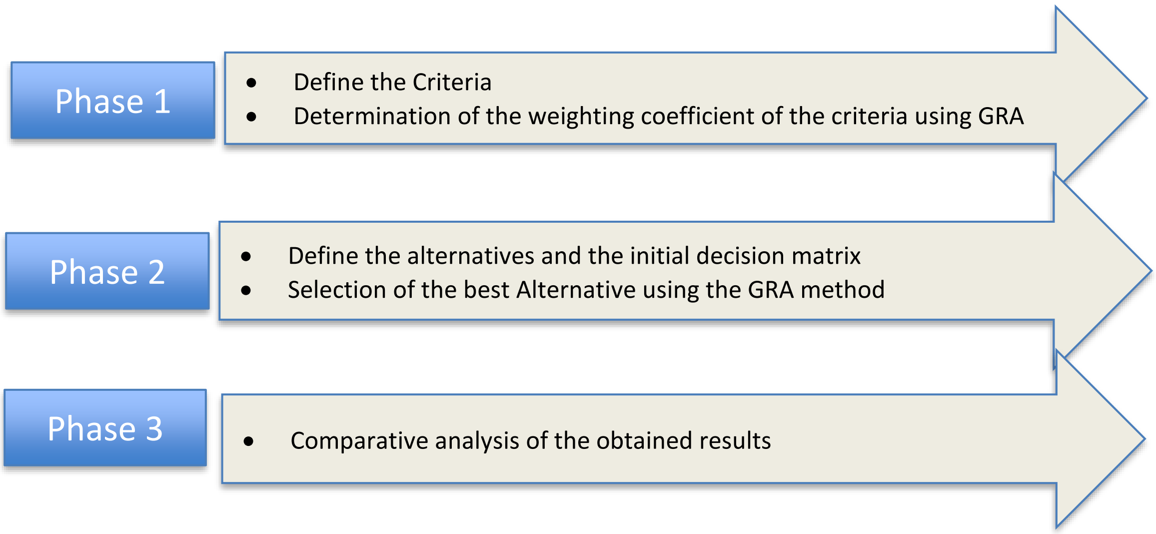

Fig. 1. Flowchart of the study.

2.2 Grey Relational Analysis (GRA) Method

The Grey Relational Analysis (GRA) [11] method is vital in decision-making and multi-criteria analysis. Originating from Grey System Theory, GRA is particularly effective when the information is incomplete or uncertain, making it suitable for handling complex relationships among multiple factors in a system. Combining GRA with PFNs enhances the ability to analyze complex systems with inherent uncertainties. For steps of GRA, see Table 1.

Table 1
Steps of the GRA Method

Step.1 Initializing the Matrix, $N = (\tilde{n}^{ij})_{m\times n}$
Step.2 Standardization of the decision matrix Cost type criteria, $CT_{ij}^{p} = \frac{\max\left(a_{ij}^{1}\right)-a_{ij}^{p}}{\max\left(a_{ij}^{2}\right)-\min\left(a_{ij}^{1}\right)}\quad z = 1,2$ Benefit type criteria, $BT_{ij}^{p} = \frac{a_{ij}^{p}-\min\left(a_{ij}^{1}\right)}{\max\left(a_{ij}^{2}\right)-\min\left(a_{ij}^{1}\right)}\quad z = 1,2$
Step. 3 Calculating the attributed weight $w_j = \frac{\sum_{j=1}^{n} H_j+1-2\times H_j}{\sum_{j=1}^{q}\left(\sum_{j=1}^{q} H_y+1-2\times H_j\right)}\ (1 \le y \le q)$
Step. 4 Calculating the PIT and NIT, $n^{p}+= \left(n_1^+, n_2^+, \ldots\ldots\ldots., n_p^+\right)$, where, n_p^+ can be evaluated using: $n_j^+ = \left(\underbrace{\max_i}_{\square} \mu_{ij}(x)\ \underbrace{\min_i}_{\square}\left(\upsilon_{ij}\right)\ \underbrace{\min_i}_{\square}\left(\pi_{ij}\right)\right)$ $n^- = \left(n_1^-, n_2^-, \ldots\ldots\ldots., n_q^-\right)$ where, each n_q^- can be calculated using the following relation: $n_q^+ = \left(\underbrace{\min_i}_{\square} \mu_{ij}(x)\ \underbrace{\max_i}_{\square}\left(\upsilon_{ij}\right)\ \underbrace{\max_i}_{\square}\left(\pi_{ij}\right)\right)$
Step. 4 Calculating the GRC of PIT and NIT $\varphi_{ij}^+ = \frac{T^+ + \varphi W^+}{d_{ij}^+ + \varphi P^+}$ $\varphi_{ij}^- = \frac{T^- + \varphi W^-}{d_{ij}^- + \varphi P^-}$
Step. 5 Calculating the GRG $\varsigma_i^+ = \sum_{j=1}^{q} w_j \varphi ij^+$ $\varsigma_i^- = \sum_{j=1}^{q} w_j \varphi ij^-$
Step.6 Calculating the weight in the GRP was: $\overline{w}_j = \frac{w_j^2}{\sqrt{\sum_{j=1}^{q} w_j^2}}$ The weight in the GRP technique for the alternative PIT and NIT was: $\wp_i^+ = \sum_j^q \left(\overline{w}_j \times \varsigma_{ij}^+\right)$ $\wp_i^- = \sum_j^q \left(\overline{w}_j \times \varsigma_{ij}^-\right)$
Step.7 Estimating the similitude to PIT and Ranking the alternatives: $RC_i = \frac{\varsigma_i^+}{\varsigma_i^+ - \varsigma_i^-}$ $RC_i = \frac{\sum_j^q\left(\overline{w}_j \times \varsigma_{ij}^+\right)}{\sum_j^q\left(\overline{w}_j + \wp_i^-\right)\sum_j^q\left(\overline{w}_j \times \varsigma_{ij}^-\right)}$

3. Basic Definitions

This section discusses basic topics, such as properties, operations, and PFNs.

Definition 3.1 [3] where X is a fixed set and a PFS P in X is an object in the form of:

P = $< x, P\left(\mu_p(x)\upsilon_p(x)\right) > |x\varepsilon X)$

where the function $\mu_p: X \to [0,1]$ is the u of the $x\epsilon X$ element to the P set and the function $\upsilon_p: X \to [0,1]$ denotes the v of the $x\epsilon X$ element to the P set. For any PFP set and $x\epsilon X$,

$0 \leq (\mu_p(x))^2 + (\upsilon_p(x))^2 \leq 1$

the function π_p was considered the hesitant point of the $x\epsilon X$ element to P set. For any PF set and $x\epsilon X$,

$\pi_p(x) = \sqrt{1 - (\mu_p(x))^2 - (\upsilon_p(x))^2}$.

For convenience, Zhang and Xu [16,17] defined PFNs as follows:

A PS set can be expressed as $P = (\mu_p, \upsilon_p)$, where μ_p and $\upsilon_p \epsilon [0,1]$ and

$H_p = (1 - (\mu_p(x))^2 - (\upsilon_p(x))^2)^{\frac{1}{2}}$ and $0 \leq (\mu_p(x))^2 + (\upsilon_p(x))^2 \leq 1$.

Definition 3.2 [5]

where, $a_p^1 = (\mu_p^i, \upsilon_p^j)$ and $b_p^1 = (\mu_p^k, \upsilon_p^l)$ are the two PFNs. Then, the arithmetic operation is as follows:

Additive property:

$a_p^1 \oplus b_p^1$ = $\left[\sqrt{(\mu_p^i)^2 + (\mu_p^k)^2 - (\mu_p^i)^2.(\mu_p^k)^2}, \upsilon_p^j.\upsilon_p^l\right]$

Multiplicative property:

$a_p^1 \otimes b_p^1$ = $[\mu_p^i.\mu_p^k, \sqrt{(\upsilon_p^j)^2 + (\upsilon_p^l)^2 - (\upsilon_p^j)^2.(\upsilon_p^l)^2}]$

Scalar Product:

$K.a_p^1$ = $\sqrt{1 - (1 - \mu_p^i)^K}(\upsilon_p^j)^K$ where K is a nonnegative constant, i.e $K \succ 0$

Definition 3.3[5] Comparison of two PFNs

where, $a_p^1 = (\mu_p^i\ \upsilon_p^j)$ and $b_p^1 = (\mu_p^k, \upsilon_p^l)$ are the two PFNs. As such, the accuracy function and the score function are as follows:

(i) Score Function: $S(a_p^1) = \frac{1}{2}(1 - (\mu_p^i)^2 - (\upsilon_p^j)^2$

(ii) Accuracy Function : $A(a_p^1) = (\mu_p^i)^2 + (\upsilon_p^j)^2$

Then, the following cases arise :

Case I: $a_p^1 \succ b_p$ iff $S(a_p^1) \succ S(b_p^1)$

Case II: $a_p^1 \prec b_p$ iff $S(a_p^1) \prec S(b_p^1)$

Case III: $if S(a_p^1) = S(b_p^1)$ and $A(a_p^1) \succ A(b_p^1)$ the $a_p^1 \succ b_p^1$

Case IV: $if S(a_p^1) = S(b_p^1)$ and $A(a_p^1) \prec A(b_p^1)$ the $a_p^1 \prec b_p^1$

Case V: $if S(a_p^1) = S(b_p^1)$ and $A(a_p^1) = A(b_p^1)$ the $a_p^1 = b_p^1$

Definition 3.4 [5], where Q and W are the two PFSs. Then, Q and W are similar sets if they hold these conditions: $\eth_Q(x) = \eth_W(x) or \mho_Q(x) = \mho_W(x)$

Definition 3.5 [5] where S and J are the two PFSs. Then, Q and W are comparable sets if they hold these conditions: ŊS(x)=£J(x) or ŊS(x)=£J(x)

Definition 3.6 [5] where Q and W are the two PFSs. Then, Q and W are equivalent sets if they hold this condition: b: €Q(x) →&W(x) b: € Q(x)→&W(x) both are bijective functions.

Definition 3.7 [5] where Q and W are the two PFSs. Then, Q is the subset of W, and W is called a superset of Q if they satisfy this condition: €Q(x) ≤ €W(x) and βS(x) ≤ βW(x)

Definition 3.8 [5], where Q and W are the two PFSs. Then, Q is called the proper subset of W it holds this condition: Q⊆ W, Q ≠W.

Definition 3.9 [5], where Q, W, and Y are the three PFSs. Then, Reflexive: Q = Q, Symmetric: Q = W and W = Q. Transitive: Q = W, W = Y and Y = Q

Definition 3.10 [5] The expected value of the PFN can be defined as follows: $I_p = \frac{1}{2}\left|1 + \mu_p^i - \upsilon_p^i - \pi_p^i\right|$ where, μ_p^i is the degree of u, υ_p^i is the degree of v, and π_p^i is the degree of hesitation.

4. Algorithm Developed using Pythagorean Fuzzy numbers (PFNs)

This section offers a solution to MADM problems based on model-based PFNs. In this approach, preferences for multiple alternatives are indicated as PFNs, and the weight vector for each criterion is determined using a heuristic algorithm. The total aggregated value is obtained by ranking the entries that use the basis similarly.

Assume that there are m alternatives for $A = (A1, A2, \dots, Am)$, and n evaluation criteria for a MADM problem. $Q = Q_1, Q_2 \dots . Q_n$ and $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ are the corresponding weight vectors for an attribute so that $\omega_j \in [0, 1]$ and the specified value, w_j is unknown such that the characteristics of each alternative are given in the form of PFNs as $\alpha_i j =< \left[a_{ij}, b_{ij}\right] >$, $(i = 1,2 \dots \dots m, j = 1,2 \dots \dots .. n)$, where $a_i j$ gives the degree in favorable terms and $b_i j$ gives the degree against the i alternative concerning the j criteria. Thus, a Pythagorean fuzzy (PF) decision matrix, to denote the decision matrix, D, can be formulated as

$$N = ([n_{ij}]_{m\times n}) \tag{1}$$

where $n_{ij}^z = \left[a_{ij}^1, a_{ij}^2\right]$. Thus, the methodology for the normalization of two distinct kinds of criteria can be given below:

Cost type criteria:

$$CT_{ij}^p = \frac{\max\left(a_{ij}^1\right) - a_{ij}^p}{\max\left(a_{ij}^2\right) - \min\left(a_{ij}^1\right)} \quad z = 1,2 \tag{2}$$

Benefit type criteria:

$$BT_{ij}^p = \frac{a_{ij}^p - \min\left(a_{ij}^1\right)}{\max\left(a_{ij}^2\right) - \min\left(a_{ij}^1\right)} \quad z = 1,2 \tag{3}$$

4.1 Calculating the Attribute Weight

There are several approaches for determining attribute weights. This section discusses the information entropy technique for calculating attribute weights. Shannon [4] established the entropy approach, similar to uncertainty, as an essential concept in thermodynamics. Entropy is a concept used in various areas, including management science and engineering science, to quantify the degree of disorder, distribution unevenness, and so on. In mathematics, entropy calculates the uncertainty and quality of useful information.

Entropy is a fundamental concept that asserts that the fuzzier a system, the higher the entropy value. So, the attribute values of all the alternatives fluctuate significantly under the attributes. In that case, the attributes will be recognized as having the greatest EP since they provide the necessary knowledge to put the alternatives in place, so they have minimal relevance in the prioritization technique. On the other hand, if the attribute values of all options differ considerably, these qualities have low entropy and help select the optimal choice.

So, while calculating the alternative, if one attribute has the highest entropy, a low weight will be assigned to it, and if one attribute has the lowest entropy, a high weight will be assigned to it.

The formulae below were used to verify the entropy value of an attribute: $E_j = -Z \times \sum_{x=1}^{n} f_{ij} ln f_{xy}$ $(1 \leq i \leq p, 1 \leq j \leq q)$, where, $= Z \frac{1}{lpq}$ and, if $f_{ij} = 0$, then $0 \times ln0 = 0$. $f_{ij} = \frac{I(r_{ij})}{\sum_{x=1}^{q} I(r_{ij})}$ $(1 \leq i \leq p), (1 \leq j \leq q)$

The entropy weight was calculated using: $\omega_j = \frac{(1-H_j)}{\sum_{x=1}^{q}(1-H_j)}$

The equation mentioned above was used to find the weight. However, Zhou et al. [4] proposed the following refined formula with which to measure the weight:

$$w_j = \frac{\sum_{j=1}^{n} H_j + 1 - 2 \times H_j}{\sum_{j=1}^{q}\left(\sum_{j=1}^{q} H_y + 1 - 2 \times H_j\right)} \ (1 \leq y \leq q) \tag{4}$$

4.2 Ranking Alternatives Based on the Developed Algorithm

A grey relational analysis (GRA) [11] is commonly used to analyze the ambiguity and incompleteness of a system model. It can produce discrete sequences with processing uncertainty, multi-variable inputs, and discrete data for correlation analysis. As a result, a GRA can be used to discuss the consistency and goal of an uncertain discrete sequence. One of the primary advantages of the grey systems theory is that it can produce good results with limited data and many variables. The grey theory is commonly used in industrial and technical decision-making.

4.2.1 Criteria of developed algorithm

The main strategy of this algorithm is to reduce the impact of various physical qualities and turn the attribute values of all alternatives into a comparable sequence. The mechanism generates a reference sequence based on those patterns, also known as the ideal option pattern or negative ideal target (NIT) sequence. This method can calculate the grey relational coefficient (GRC) between comparability sequences and the reference sequence. Finally, using these GRCs, the grey relational grade (GRG) here between a known segment and each such considered comparability sequence is completely determined. If a comparability sequence transformed with an alternative option has the maximum GRG with the reference sequence, it is the primary target sequence; otherwise, it is the critical ideal target sequence. Assuming the decision-making matrix has been normalized, as previously described, then the stages of the grey relational projection (GRP) technique will be completed as follows:

4.3 Positive Ideal Target and Negative Ideal Target

If a normalized PFN decision matrix has been determined, then the PFNs PIT and the corresponding PFNs NIT can be defined as follows concerning the definition of PIT: $n^{p}+= (n_1^+, n_2^+, \dots\dots\dots, n_p^+)$, where, n_p^+ can be evaluated using:

$$n_j^+ = \left(\underbrace{\max_i}_{\square} \mu_{ij}(x) \underbrace{\min_i}_{\square} (v_{ij}) \underbrace{\min_i}_{\square} (\pi_{ij}) \right) \tag{5}$$

the PFNs can also be evaluated using: $n^- = (n_1^-, n_2^-, \dots\dots\dots, n_q^-)$, where, each n_q^- can be calculated using the following relation:

$$n_q^+ = \left(\underbrace{\min_i}_{\square} \mu_{ij}(x) \underbrace{\max_i}_{\square} (v_{ij}) \underbrace{\max_i}_{\square} (\pi_{ij}) \right) \tag{6}$$

Fuzzy PIT can also be defined as: $n_j^+ = ([1,1])$ and $n_j^- = ([0,0])$

4.3.1 Calculating the GRC

The GRC of each PIT and NIT alternative form can be calculated using the equations below. Each alternative GRC form of PIT is given as: $\varpi_{ij}^{+} = \frac{T^{+}+\varphi W^{+}}{d_{ij}^{+}+\varphi P^{+}}$, where the normalized hamming distance is: $D_{ij}^{+} = \frac{1}{2}\left[\left|\mu_{ij}^{2} - \mu_{j}^{+}\right| + \left|\upsilon_{ij}^{2} - \upsilon_{j}^{+}\right| + \left|\pi_{ij}^{2} - \pi_{j}^{+}\right|\right]$, $T^{+} = \underbrace{\min}_{i}\underbrace{\min}_{j} d_{ij}^{+}, W^{-} = \underbrace{\max}_{i}\underbrace{\max}_{j} d_{ij}^{+}$, φ is the resolution coefficient, and its values form this range, $\varphi\varepsilon(0,1)$.

So, the GRC of each alternative in the OIS is given as:

$$\varphi_{ij}^{-} = \frac{T^{-}+\varrho W^{-}}{d_{ij}^{-}+\varphi P^{-}} \tag{7}$$

where, the normalized hamming distance is: $D_{ij}^{-} = \frac{1}{2}\left[\left|\mu_{ij}^{2} - \mu_{j}^{-}\right| + \left|\upsilon_{ij}^{2} - \upsilon_{j}^{-}\right| + \left|\pi_{ij}^{2} - \pi_{j}^{-}\right|\right]$, $T^{-} = \underbrace{\min}_{i}\underbrace{\min}_{j} d_{ij}^{-}, W^{-} = \underbrace{\max}_{i}\underbrace{\max}_{j} d_{ij}^{-}$, φ is the resolution coefficient, and it value is form this range $\varphi\varepsilon(0,1)$

So, the GRC of each alternative in the PIS is given as:

$$\varphi_{ij}^{-} = \frac{T^{-}+\varphi W^{-}}{d_{ij}^{-}+\varphi P^{-}} \tag{8}$$

4.3.2 Calculating the GRG

The GRG of each PIT and NIT alternative form can be calculated using the equations below.

$$\varsigma_{i}^{+} = \sum_{j=1}^{q} w_j \varphi ij^{+} \tag{9}$$

$$\varsigma_{i}^{-} = \sum_{j=1}^{q} w_j \varphi ij^{-} \tag{10}$$

4.4.3 Ranking:

The GRA ranking method involves selecting the alternative "greatest degree of grey relation" form of PIT and the "smallest degree of grey relation" form of PIT from a set of NIT options.

4.3.4 Projection:

The weight in the GRP was:

$$\overline{w}_j = \frac{w_j^2}{\sqrt{\sum_{j=1}^{q} w_j^2}} \tag{11}$$

The weight in the GRP technique for the alternative PIT and NIT was:

$$\wp_{i}^{+} = \sum_{j}^{q}\left(\overline{w}_j \times \varsigma_{ij}^{+}\right) \tag{12}$$

$$\wp_{i}^{-} = \sum_{j}^{q}\left(\overline{w}_j \times \varsigma_{ij}^{-}\right) \tag{13}$$

4.3.5 Estimating the similitude to PIT and rating the alternatives:

The option under consideration may now be positioned based on the GPC projection coefficient of each alternative based on the PIT and NIT solutions. The greater the projected number on the PIT, and the closer to the NIT, the better will be the alternative, whereas on the contrary, the smaller the projection onto the PIT, and the farther away from the NIT, the better will be the alternative. The similitude estimation can be defined as follows:

$$RC_i = \frac{\varsigma_i^{+}}{\varsigma_i^{+} - \varsigma_i^{-}} \tag{14}$$

$$RC_i = \frac{\sum_{j}^{q}\left(\overline{w}_j \times \varsigma_{ij}^{+}\right)}{\sum_{j}^{q}\left(\overline{w}_j + \wp_{i}^{-}\right)\sum_{j}^{q}\left(\overline{w}_j \times \varsigma_{ij}^{-}\right)} \tag{15}$$

5. Study Case

Service levels in numerous businesses have risen in the context of the globalization of the economy. The national economy is expanding, and people's needs are increasing, thus promoting the rapid expansion of tourism. As a result, they must anticipate future market trends and compete with cutting-edge, high-quality services. The key theme of inquiry was how to make good use of hotel management and customer consumer psychology to develop a realistic service quality evaluation system that is useful in practice to assist firms in enhancing their competitiveness in terms of the quality of service.

With regard to MADM issues for service suppliers, four major international hotels were selected—namely, Emirates Palace, Rancho Valencia Resort Spa, The Westin Excelsior, and Burj Al Arab. Four criteria were used to determine the optimal service supplier: Good customer service, Location, Hygiene, and Quality of cuisine.

The assumption of the alternatives A_1, A_2, A_3, A_4 would be in the form of the PFNs w.r.t four attributes, Q_1, Q_2, Q_3, Q_4, for the DM to be formed (see Table 2).

Table 2
Attributive values of the alternatives.

Alternative⇒	Q_1	Q_2	Q_3	Q_4
A_1	(0.1,0.6)	(0.2,0.4)	(0.1,0.7)	(0.8,0.1)
A_2	(0.5,0.4)	(0.6,0.3)	(0.2,0.5)	(0.1,0.3)
A_3	(0.1,0.5)	(0.3,0.5)	(0.4,0.3)	(0.2,0.6)
A_4	(0.2,0.7)	(0.2,0.3)	(0.2,0.1)	(0.8,0.2)

The computational steps of the application of the herein-proposed methodology for the problem at hand were as follows:

Step I:

The following matrix (see Table 3) was standardized using Equations 1, 2, and 3: $N = (\tilde{n}^{ij})_{m\times n}$

Table 3
Attributive values of the alternatives

Alternative⇒	Q_1	Q_2	Q_3	Q_4
A_1	(0,0.625)	(0.125,0.375)	(0,0.75)	(0.875,0)
A_2	(0.5,0.375)	(0.625,0.25)	(0.125,0.5)	(0,0.25)
A_3	(0,0.5)	(0.25,0.5)	(0.375,0.25)	(0.125,0.625)
A_4	(0.125,0.75)	(0.125,0.25)	(0.125,0)	(0.875,0.125)

Step II:

The notion of expected value arose, and it was defined as the central value of the interval. The expected value from the entropy weight was calculated using Equations 4, 5, and 6.

$$I_n^p = \begin{bmatrix} 0.145 & 0.045 & 0.1535 & 0.5545 \\ 0.166 & 0.2795 & 0.071 & 0.074 \\ 0.1301 & 0.03 & 0.12 & 0.087 \\ 0.092 & 0.016 & 0.065 & 0.5175 \end{bmatrix}$$

The attribute weights were: $\chi_1 = 0.1556$, $\chi_2 = 0.2816$, $\chi_3 = 0.2876$, $\chi_4 = 0.2876$

Step III:

As PIT maximizes the benefits criterion and minimizes the cost criterion while NIT minimizes the benefits criterion and maximizes the cost criterion of the PFNs, they were verified using Equations 7 and 8 :

$$\varsigma_j^+ = [(0.5,0.625),\ (0.25,0.375),\ (0.125,0.75),\ (0.125.0)]$$

$\varsigma_j^- = [(0,0.5),\ (0.625,0.5),\ (0,0.5),\ (0.875,0.25)]$

Step IV:

The GRC was verified to determine the correlation between the ideal and the actual values. The GRC for the PIT and the NIT were verified using Equations 9 and 10:

$$(\varsigma_{ij}^+)_{4\times4} = \begin{bmatrix} 0.2702 & 0.9677 & 0.769 & 0.118 \\ 0.289 & 0.227 & 0.243 & 0.61 \\ 0.197 & 0.465 & 0.166 & 0.1969 \\ 0.275 & 0.476 & 0.139 & 0.1147 \end{bmatrix}$$

$$(\varsigma_{ij}^-)_{4\times4} = \begin{bmatrix} 0.2845 & 0.099 & 0.281 & 0.102 \\ 0.133 & 0.145 & 0.225 & 0.1196 \\ 1 & 0.139 & 0.239 & 0.062 \\ 0.143 & 0.086 & 0.180 & 0.526 \end{bmatrix}$$

Step V:

After calculating the GRC, the weight of the GRP of the alternative A_i on the PIT and NIT were calculated using Equations 11, 12, and 13.

$P_1^+ = 0.2945,\quad P_2^+ = 0.18154\ ,\quad P_3^+ = 0.13564,\quad P_4^+ = 0.1142$

$P_1^- = 0.08754,\quad P_2^- = 0.0818\ ,\quad P_3^- = 0.114682\ ,\quad P_3^+ = 0.13324$

StepVI:

The relative closeness was calculated using equation 14.

$RC_1 = 0.7709,\quad RC_2 = 0.6893,\quad RC_3 = 0.54203,\quad RC_4 = 0.4616$

Step VII:

The alternatives were ranked according to their estimated similitude to each PIT using equations 14 and 15: $N_4 > N_3 > N_2 > N_1$.

6. Comparative Analysis

Traditional fuzzy methods have been a staple in decision-making processes, especially where uncertainty is a significant factor. However, these methods are inherently limited by their linear constraints. These constraints often lead to less accurate and less flexible representations of uncertainty, compromising the quality of the decisions derived from these methods. Picture Fuzzy Numbers (PFNs) represent a significant advancement in this area. Unlike traditional fuzzy sets, PFNs incorporate three degrees of membership: positive, neutral, and negative. This additional granularity allows for a more detailed and nuanced representation of uncertainty, thus improving the accuracy and robustness of the decision-making process.

In the realm of Multi-Attribute Decision Making (MADM), Grey Relational Analysis (GRA) and Grey Relational Projection (GRP) techniques stand out when compared to other MADM methods. One of the key strengths of GRA and GRP is their ability to handle incomplete and uncertain information effectively. Traditional MADM methods often require precise and complete data, which is not always available in real-world scenarios. GRA and GRP, however, are designed to work with the available information, no matter how incomplete or uncertain. Integrating PFNs with GRA and GRP techniques further enhances their capability to manage complex decision-making scenarios. Using PFNs, GRA and GRP can process and analyze data more flexibly and in detail, leading to more accurate and reliable decision outcomes.

Weighting attributes also play a crucial role in the decision-making process. The entropy method offers a sophisticated approach to determining the importance of each attribute based on the variability of the data. Entropy, in this context, measures the degree of disorder or uncertainty in the data. Attributes that exhibit higher variability are considered more significant and are assigned higher weights. This starkly contrasts the equal weighting method, which assumes that all attributes are equally important regardless of their actual impact on the decision. By systematically determining weights based on data variability, the entropy method ensures that the weighting reflects the true significance of each attribute, leading to more informed and balanced decision-making.

To elaborate further, traditional fuzzy methods typically handle uncertainty by assigning a single membership value to each element, which simplifies the uncertainty but can also lead to loss of critical information. PFNs overcome this limitation by allowing for a spectrum of membership values, simultaneously capturing the degrees of agreement, neutrality, and disagreement. This multidimensional approach to uncertainty provides a richer and more comprehensive view of the decision problem.

GRA and GRP techniques, particularly when combined with PFNs, are adept at dealing with the complexities inherent in real-world decision-making scenarios. For example, when data may be sparse or imprecise, these methods can still derive meaningful insights by focusing on the relative relationships between different options rather than requiring absolute precision. This makes them particularly useful in environmental management, medical diagnostics, and any domain where data quality cannot be guaranteed.

The entropy method's ability to dynamically adjust attribute weights based on data variability adds another layer of sophistication to the decision-making process. Unlike the equal weighting method, which can oversimplify the importance of different attributes, the entropy method recognizes that not all attributes contribute equally to the decision outcome. By quantifying the inherent uncertainty and variability in the data, the entropy method ensures that the decision-making process is grounded in the actual significance of the available information.

7. Conclusions

The information on the score values observed for Multi-Attribute Decision Making (MADM) issues is often inexplicit, vague, untrustworthy, and conflicting. To address this challenge, Picture Fuzzy Numbers (PFNs) have emerged as a valuable tool for gathering and processing this type of information during the MADM process. This article delved into the application of PFNs in evaluating MADM problems by scrutinizing rating values.

The study began with a detailed description of the expected value, entropy, and Hamming distance within the context of the Grey Relational Analysis (GRA) technique for MADM using PFNs. Initially, the Positive Ideal Target (PIT) and Negative Ideal Target (NIT) were computed to establish benchmarks. This was followed by the computation of the Grey Relational Grade (GRG) between each option and the ideal alternative based on the PIT and NIT values.

Subsequently, the relative relational degree was determined by evaluating the GRG relative to the PIT and the NIT simultaneously to achieve a comprehensive ranking of all the options. This method ensured a more balanced and nuanced assessment of the alternatives.

Illustrative examples were provided to demonstrate the practicality and effectiveness of the proposed strategy. These examples highlighted how the technique could be applied in real-world scenarios, reinforcing its utility. Furthermore, the proposed method was compared with other existing approaches to establish its transparency and effectiveness in handling diverse decision-making challenges within the PFN framework.

The study results showed that the proposed technique is effective and versatile, making it suitable for various decision-making problems. It is recommended that this technique be extended to other domains. Potential applications include medical diagnostics, where precise and reliable decision-making is crucial, and improving cleaner production processes in industries such as gold mining, where environmental and operational efficiency are key concerns.

In conclusion, using PFNs in the MADM process provides a robust framework for addressing the inherent uncertainties and ambiguities in decision-making. By leveraging the expected value, entropy, and Hamming distance within the GRA technique and computing the GRG in relation to the PIT and NIT, this study has laid the groundwork for more transparent and effective decision-making strategies. Future research should continue exploring and expanding this technique's applications, ensuring its benefits are realized across various fields and industries.

Author Contributions

The study was conceived and designed by E.R., N.K., S.S.S.A., N.A., and O.A..O, who contributed equally. All authors equally participated in data collection, interpretation, methodology, writing, and editing. All authors have read and approved the final manuscript.

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Conflicts of Interest

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