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Deriving the Classical and New-Keynesian Phillips Curves using Machine Learning Simulations

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ABSTRACT

This study investigates inflation dynamics through the lens of pricing strategies in a simulated market economy, where firms set prices reflecting their expectations of future demand. Building on the foundational work of Milton Friedman, Robert E. Lucas, we propose a novel approach that considers price-setting as an emergent property of limited rationality among firms. Using a simulation with N participants acting as firms, we examine the implications of five distinct pricing strategies—"Trial-and-error", "Tit-for-tat," "Forgiving," "Competitive," and "Random"—over 12 rounds, representing a financial year. The simulation incorporates a midgame demand shock, revealing how firms adapt their pricing decisions. Results demonstrate that limited rationality leads to short-run deviations from equilibrium, aligning with the expectations-augmented Phillips curve, and eventually converging to a long-run equilibrium, consistent with the classic Phillips curve. Our findings show that price adjustments, rather than demand increases, drive production in the long run, confirming the theoretical vertical Phillips curve. In contrast, short-run outcomes exhibit heterogeneity and a gradual convergence of prices following demand shocks. By simulating these games 10,000 times across varying participant numbers, we highlight the role of imperfect information and expectation formation in shaping economic outcomes. This study bridges microeconomic behaviour with macroeconomic theory, offering insights into the interplay between rational expectations and limited rationality in determining inflation and employment.

1. Introduction

Inflation, as defined in contemporary economic literature, refers to the rise in the general level of prices within an economy. It is widely believed that inflation has a short-term negative correlation with the utilization of economic resources, particularly employment. This relationship was famously addressed by the Friedman-Lucas critique, which played a pivotal role in shaping modern economic thought. Both Friedman [1] and Lucas [2; 3; 4] introduced the expectations-augmented Phillips curve, marking a significant departure from earlier interpretations. Building on their foundational work,

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further refinements were made by Barro and Grossman [5], Gordon [6], Calvo [7], and later, Smets and Wouters [8] culminating in what is now known as the New-Keynesian model. Many of the iterations of this model are based on the assumption that firms are either well-informed and make rational pricing decisions (as articulated by Phelps [7; 10], Friedman [1], Sargent [11] and Hall [12] or, alternatively, that firms operate with imperfect information (a position taken by Samuelson et al. [13] and Calvo [7]. In this study, we propose a different approach, arguing that prices can be seen as objective measures of the subjective choices made by participants within the economy. Consequently, prices act as a form of "voting" by economic agents on the likelihood of future adverse or favourable shifts in demand. To explore this hypothesis, we employed a simulation that reveals evidence supporting the law of limited rationality, which emerges from the optimum-seeking behaviour of economic participants. In our model, various pricing strategies were analysed to determine which one follows the most optimal path for price discovery. Besides the trial-and-error method, we introduced four additional pricing rules: "Tit-for-tat," "Forgiving," "Competitive," and a control rule, "Random." These rules were tested against the majority decisions made by producers within the simulation. To ensure the robustness of our results, the experiment was repeated 10,000 times, and the outcomes were compared with actual economic data from the United States and Hungary. Through this simulation, we derived the equations for both the classic and expectations-augmented New-Keynesian Phillips curve. Our findings suggest that the pricing strategies employed by participants play a crucial role in shaping the dynamics of inflation and employment, providing deeper insight into the decision-making processes that govern economic outcomes.

2. Literature Review

Classical economic theory suggests that the rate at which the price of a commodity or service increases corresponds to the size of the excess demand for that commodity or service. Following this logic, a similar relationship could be inferred between employment and the price of labour. When the demand for labour is high, unemployment tends to be low, as employers are expected to compete for available workers by bidding up wages. However, the reverse does not hold as strongly; when labour demand is low, wage prices adjust downward at a much slower pace. This phenomenon, referred to as "sticky wages," aligns with Keynesian economics, as highlighted by [14; 15; 16]. To gain deeper insights into companies' pricing decisions, game theory offers valuable tools. In Dynamic Stochastic General Equilibrium (DSGE) models, solutions are typically found at equilibrium, a point consistent with the rational expectations theory where any system naturally converges. This aligns with the Nash equilibrium in game theory, where no participant can improve their strategy without disadvantage. However, experimental evidence, such as the work of [17; 18] shows demonstrates the prevalence of limited rationality, even in controlled environments where participants possess knowledge about future demand. Our simulations further support this conclusion. In our simulations, we recreated a controlled laboratory environment with several key conditions: (1) a known, finite number of participants, and (2) the relaxation of assumptions regarding the effects of money creation by banks. Simulations offer advantages over traditional laboratory experiments, such as cost-effectiveness (no setup costs or financial rewards needed for participants), flexibility in controlling decision-making (e.g., encoding pricing strategies), and reproducibility (experiments can be repeated indefinitely). Unlike real-world experiments, simulations eliminate time constraints and participant fatigue. In our model, N students act as producers in a market economy. Unlike in a Kydland-Prescott environment, there are no external observers or central banks. The simulation assumes a free-market condition where demand for production is fully realized.

Suppose that there are N students in this experiment acting as producers. There are no external observers and no central bank, unlike in a Kydland-Prescott environment. The conditions are close to those persistent in a theoretical free market. The simulation allows for the full realization of the supply, in other words, the demand for the produce is guaranteed. We programmed the game, so that given a demand equation as well as a cost equation, the goal of each participant is to maximize the profit function (derived as revenues minus costs) in each of the 12 rounds, while an upward shift in demand occurs midway through the game. A separate demand function that incorporates the price as well as an external demand variable is used to estimate the revenues of the firms. In essence, there are two demand functions which the players were encoded to optimize. Between rounds 1 – 6: $y = \omega_1 - p$, and between rounds 7 – 12: $y = \omega_2 - p$. Needless to say, it is specifically this midgame demand uptick, where the price adjustment can be observed. The output of the simulation is a time series of prices set by each of the N students in the 12 rounds of the game. The external demand variable ω changes from a value ω_1 to a valueω_2, where $\omega_2 > \omega_1$ indicating a surprise uptick in demand. It will be shown in this study that ω represents the output gap, or excess capacity. By substituting the external demand variable ω to the output gap (or output potential) $\tilde{y}$ we get the classic Phillips curve. The aim of this study is to replicate the findings of some laboratory experiments published by [17; 18] and derive the classic as well as the New-Keynesian Phillips curve.

We realize the limitations of this study, namely, the external variable ω is known for the whole duration of the game. In economic realities, of course, ω is not known in advance, producers of goods and services do not know what the output gap will be in the future, they can only estimate it based on the limited current information they possess. We use machine learning algorithms to simulate the results of a laboratory experiment with N participants who are in the role of producers in a market economy. Thanks to the second benefit of using AI, we can encode all or some of the players to follow one of two pricing methodologies: the trial-and-error method, or a rule methodology. In the trial-and-error method, the first price is set randomly and every subsequent price choice is adjusted as a reaction to the success or error of the previous one, so that $p_{n+1} = p_n + \Delta p$, where Δp is the change applied to the current guess based on the results of the previous trial. The error is minimized by $|f(p_n)| < \varepsilon$, where ε is a small positive number representing the error margin.

Table 1
Rule-Based Pricing Method for the Two Games

Classic Phillips Curve Simulation (Game 1)	Calvo-Pricing Model Simulation (Game 2)
Tit-for-Tat - the $\frac{N}{4}$ subset of participants raises prices if in the previous round, the group raised them. $p_{t+1} = P_{t-1}$, where p the price set by the subset of participants is, P is the average prices set by the group in round$t - 1$.	Tit-for-Tat - the $\frac{N}{4}$ subset of participants raises prices if in the previous round, the group raised them. The price adjustment equation is the same as in the Classic Phillips curve simulation. The group raises prices with a probability $\frac{1}{3}$
Forgiving - the $\frac{N}{4}$ subset of participants raises prices only in the case that the group raises them for two consecutive rounds. The price is adjusted following the equation:$p_{t+1} = aP_{t-1} + \mu$, where and μ is an error term.	Forgiving - the $\frac{N}{4}$ subset of participants raises prices only in the case that the group raises them for two consecutive rounds. The price adjustment equation is the same as in the Classic Phillips curve simulation. The group raises prices with a probability $\frac{1}{3}$
Competitive - the $\frac{N}{4}$ subset of participants raises prices regardless of the groups' decision. The price is adjusted upward according to$p_{t+1} = aP_{t-1} + bE(P) + \mu$, where a and b are adjustment parameters, $P_{t+1}$is the average price set by the group in the previous period, $E(P)$ is the expected price set by the group in round $t + 1$ and μ is an error term.	Competitive - the $\frac{N}{4}$ subset of participants raises prices regardless of the groups' decision. The price is adjusted in the same manner as in the Classic Phillips curve simulation. The rest of the group raises prices with the probability $\frac{1}{3}$
Random - the $\frac{N}{4}$ subset of participants raises prices with a probability$\frac{1}{2}$.	Random - the $\frac{N}{4}$ subset of participants raises prices with a probability $\frac{1}{2}$. The rest of the group raises prices with the probability $\frac{1}{3}$

However, just like in the case of the students participating in a laboratory experiment, they might not necessarily adopt a trial-and-error approach throughout the game. They might adopt a strategy based on their preconceptions about market behavior. We reflected this in our simulation.

The rules we encoded are similar to those used in a War Game tournament designed by Robert Axelrod that modeled retaliation strategies between two superpowers during the cold war [19]. The rules were created around the framework of the prisoner's dilemma. If the player is encoded to follow a rule, then the first price the player gives is not random (unless they follow the "Random" rule), rather, it is precisely determined by a mathematical relationship between the price P set by the group in the previous round and the price p the player set in the previous round, sometimes augmented with future expectations. Game 1 will be simulated in two versions. One is a non-cooperative game, where there is no prize for a second place, only one player is chosen as the winner. Each player maximizes their own profit function with the only view to set a price in the subsequent round consistent with further maximizing the function, and not to overshoot or undercut it. Cooperation is therefore not present in the game. A second version is where the goal is for the group to maximize the revenues. But if we consider the question from the side of a consumer, there might be a different goal for the direction of prices. The consumers (price takers) might prefer a lower price that may be below ω_1 or ω_2. In that case, they might cooperate and refuse to raise prices at a very high rate. To reflect the effects of cooperation of price takers and price givers, we set up a rule-based pricing determination method to allow for cooperation among a subset of N participants in the game. The rules are arranged in Table 1 in an ascending order from simplest to the more complex.

The group means $\frac{3N}{4}$ in each case, and the adjustment parameters differ from one another according to the rule.

The aim of the study can be summarized in the following two points:

First, it is to determine if a rule-based price setting methodology by the subjects of the game provides any short or long-term advantage in winning the game (maximizing the profit function).

Second, to derive the equations that dictate the limited rationality driven pricing decisions of the players – the long-run and short-run Phillips curve.

In our simulation, participants, like students in laboratory experiments, did not uniformly follow a trial-and-error approach throughout the game. Instead, they adopted strategies shaped by preconceived ideas about market behaviour. This variability in strategy was accounted for in our simulation through the encoding of rules that resemble those used by Robert Axelrod in his famous War Game tournament, which modelled Cold War-era retaliation strategies between superpowers [19]. These rules, derived from the prisoner's dilemma framework, were applied to simulate strategic decision-making in the context of pricing behaviours. The simulation was executed in two distinct versions. In the first, a non-cooperative game was modelled, where each participant focused on maximizing their own profit, without any consideration for the collective welfare of the group. In this version, only one participant was declared the winner, as there was no prize for second place. Each player was incentivized to optimize their pricing strategy for individual gain. This non-cooperative scenario closely mirrors competitive market environments, where firms are driven by profit-maximizing behaviour. The second version introduced a cooperative element, where participants worked together to maximize group revenues. This cooperative approach modelled a scenario where the collective outcome of the group was prioritized over individual profits. Such cooperation could be viewed as a form of implicit collusion, which might arise in real markets under certain conditions, particularly when firms aim for price stability. From a consumer's perspective, this cooperative behaviour could lead to more stable or lower prices, reflecting a different dynamic compared to the non-cooperative game. In this case, price stability might be a preferable outcome, as consumers generally favour lower prices, and cooperation among producers could result in more moderate price increases. To analyse the pricing strategies employed in both the non-cooperative and cooperative versions, we introduced a rule-based pricing determination method. This allowed for varying degrees of cooperation among a subset of participants, reflecting a range of behaviours from full competition to partial collusion. The rules used in the simulation were ranked in Table 1, starting with the simplest strategies and progressing to more complex ones. This framework enabled a comprehensive analysis of how different levels of cooperation and competition influenced pricing behaviour, with a particular focus on the interaction between price takers (consumers) and price setters (producers). By examining the outcomes under both cooperative and non-cooperative conditions, the simulation provided valuable insights into the impact of strategic interactions on market prices. The rules-based approach allowed for a deeper understanding of how cooperation among producers can affect market dynamics, particularly in relation to price stability and consumer welfare.

3. Database and Methods

An overlooked paper authored by American statistician Irving Fisher published nearly 30 years before Phillips' work already outlined the relationship between unemployment and inflation using the monthly index of wholesale prices and the index of trade of [20]. In his work, he concludes that (1) not the price relates with trade, but the rate of change of the price, (2) inflation causes changes in trade and not the other way around, and finally (3) The correlation between the price change and trade are high when lag of 9.5 months is applied to the trade variable. In his seminal 1958 paper, Phillips found statistical evidence to support the hypothesis stated in the beginning of the work that the rate of change of money wage rates (inflation of wages) can be explained by the level of unemployment, except in case where import prices offset the increase in productivity [21]. Phillips fitted the following equation to the data on Britain from 1861- 1957.

$$\log(\pi + a) = \log(b) + c\log(u)$$

Or

$$\pi = -a + bu^e$$

Where π is inflation, u is the unemployment rate, b, c are constants estimated by an OLS, and a is an adjustment parameter to fit the curve to extreme values. The estimates for the shape of the original Phillips curve are as such: $a = 0.9$, $b = 9.638$, e= -1.394. Samuelson and Solow [13] refined the non-linear relationship, replaced the change of price of wages with change of prices, and thanks to their prominence in the field of economic thought, their conclusion that the unemployment rate can be lowered by a rise in inflation has gained traction among the FED officials. Up until the 1960s the Phillips Curve was an equation containing inflation, unemployment rate, some measure of expected inflation based on lags and taxes. Lipsey [22] formulation added the cost of living as an additional explanatory variable, however it refutes the cost-push (wage-price) spiral. He also noted that there is strong feedback from price changes to wage changes. What Lipsey also found was that in the observed datasets the Phillips curve was gradually shifting to the right.

The natural unemployment rate revolution came with Friedman [1] presidential address. The natural rate is the equilibrium point of unemployment in any economy. There is always some level of unemployment that is due to lack of skills or lack of capital [9]. The transition of the Phillips curve from a downward-sloping curve to a vertical line occurs due to the role of inflation expectations. When employment is below the natural rate, the short-run Phillips curve initially shifts upwards as inflation expectations adjust, and the higher inflation manifests. When the adjustment happens, there is no trade-off between inflation and unemployment in the long run [23]. The assumptions of Friedman and Phelps are continuous market clearing and imperfect information (in Friedman's model the company is more knowledgeable than the consumer, while in Phelps', both the consumer and the company are not knowledgeable)

The model's initial form is the same as Lipsey's, however it features an expectation augmented term. The real wage inflation is defined as nominal wage inflation less the rate of price inflation.

$$w = f(u - u^*) + \pi^e$$

Where w is nominal wage inflation; u is actual unemployment rate; and u^* is the natural rate of unemployment, π is rate of price inflation. The Friedman-Phelps' assumption is that labor is the only cost of production and there is no productivity growth, hence the following relationship holds:$\pi = w$. Substituting in the above, we imply that the long-run Phillips curve is determined only by labour supply and demand, which is independent of inflation [4]. This is where the law of rationality emerges. In order for the above statement to be true, the expectation of inflation must manifest, that is $\pi = \pi^e$, and consequently,$u = u^*$.

$$\pi = f(u - u^*) + \pi^e$$

Turning to the Fisher-Cagan equation to confirm the Friedman-Phelps accelerationist inflation position, we come to the following

$$w = a\sum_{i=1}^{m} v_i \pi_{t-i} + f(u - u^*) + \mu$$

Why at all should the sum of the weights be $\sum_{i=1}^{m} v_i = 1$? Consider the following thought experiment let's say inflation had been at two percent for a long period of time, and it went up to three percent per annum, staying there permanently. It is then reasonable to expect that people will make rational expectations that inflation will continue at 3 percent per annum. This means that the weights of the else than the actual rate of inflation, that is 3%, are zero, and the rest are one [25]. A consistent inflation will soon be perfectly anticipated. But in reality, inflation never follows the path described in the thought experiment. When we refer to a "rational" expectations hypothesis, all of the weights v_i in the inflation expectations equation amount to 1. Sargent [25] stated, that if the weights sum or are close to unity, the inflation rate would be serially correlated. If the v_i's are overestimated, it means tat the a's are underestimated leading to spurious correlations.

$$\pi_t^e = \sum_{i=1}^{m} v_i \pi_{t-i} + \mu$$

The reasoning is consistent with the neo-classical theory, that real wages are determined by labor demand and supply, ceteris paribus [26]. On a note from the authors, which we shall later expand on, this does not necessarily mean that other effects not captured by the model can't affect the relationship. Quite, the contrary, a rise in the money supply, especially if it is not backed by a rise in productive capacity can create great imbalances and the distance between true market equilibrium and the market's price mechanism can increase. This is an aspect that modern economic schools seldom mention. We won't go into the details of it in this paper. Lucas [3; 4] augmented the model of Phelps and Friedman by allowing for rational expectations instead of adaptive expectations. This cemented the long-run Phillips curve into academia. According to an IMF working paper by Palley [27], Palley (2011), the Lucas-Phelps-Friedman critique formulated two great findings: (1) adaptive/rational expectations, (2) no permanent trade-off between inflation and unemployment. From that point onward (At least until the 1980s) the discourse on the possibility of full employment waned and turned into a discourse about the natural rate of employment. From that point onwards, an unravelling in American politics began with weakening institutions, worker unions and worker protections. The Lucas-Phelps-Friedman natural rate hypothesis therefore transformed conventional view on the inefficiency of monetary policy.

In this research on the history of the Phillips curve, Gordon [28] noted that the trio's conclusions on the long-run relationship can't be reconciled with the theory of real business cycles. For the trios' theory to be correct, business cycles need to be significantly shorter. In between the 1960s and the 1980s, no distinct correlation could be found between unemployment and inflation (measured by the

PCE). The evolution of Gordon's models came in a few steps:

First, Gordon tried to predict inflation using a multifactor model [29]:

$$g_{gpt} = g_{\left(\frac{w}{q'}\right)_L} + g_{\left(\frac{w}{q}\right)_t} + g_{\left(\frac{o}{s}\right)_t} + g_{mt}$$

$$g_{\left(\frac{w}{q'}\right)_t} = a + m_t + g_{c*t} + g_{QL} + g_{Tst}$$

$$\pi_t^e = \sum_{i=1}^{m} v_i \pi_{t-i}$$

Where g_{gpt} is the rate of growth of non-private farm deflator, $g_{\left(\frac{w}{q'}\right)_L}$ is the rate of wage divided by productivity at full capacity, $g_{\left(\frac{w}{q}\right)_t}$ wage divided by productivity, $g_{\left(\frac{o}{s}\right)_t}$ is the ratio of new orders to shipment, g_{mt} and mt are total employment rate of hours per man and its rate of growth in the entire economy. g_{c*t} is the expected rate of change in the consumer price index, g_{QL} is the rate of growth of output, g_{TSt} is the rate of growth of the social security tax rate. π_t^e is expected inflation and v_i are weights on past inflation. The sum of the weights is unity Goutsmedt and Rubin [30]. Sargent criticized the logic of constructing the approximation of the expected inflation, as it had no theoretical or econometric justification [25].

The model allowed for a long-run non-vertical relationship between inflation and unemployment. However, it couldn't explain the stagflation that came in the 1970's, but it supported the "full employment policy" (something refuted by the Lucas-Phelps-Friedman critique).

Second, as a response to the realities of stagflation, Gordon [31] incorporated the natural rate hypothesis into the model. What came is a refined and rethought model that defied Lucas' symmetry in periods when unemployment is above the natural rate for prolonged periods of time, as Lucas' model predicted accelerating deflation. Gordon's model involved a threshold [31]:

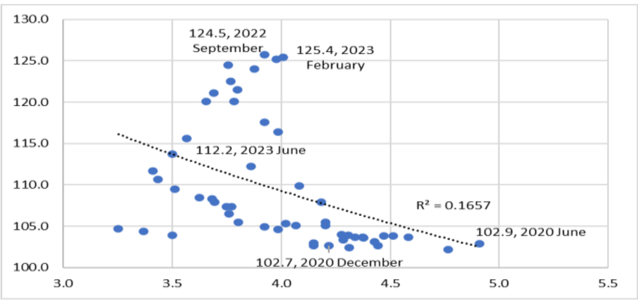
$$g_{w_t} = aX_t + b\left(g_{d_t^e}\right)g_{d_t^e}$$

$$b_t = c g_{d_t^e}, 0 \leq g_{d^e} < \frac{1}{c}$$

Where $g_{w_t}$the elasticity of wage change is, aX_t are variables multiplied by respective coefficients, $g_{d_t^e}$ is the expected rate of inflation. C is the consumer price index.

The model states that from a point of low inflation corresponding to a high unemployment rate (the deviation from the natural rate is short in time), the elasticity of wage change b will be low, however, as the labor market tightens, there will be accelerating inflation, and not deflation, as predicted by [2]. In other words, when unemployment is higher than the natural rate, inflation will accelerate higher. This is because higher wages cause inflation, which increases the expected rate of inflation, and in turn raises the elasticity of wage change to expected inflation.

A.



B.

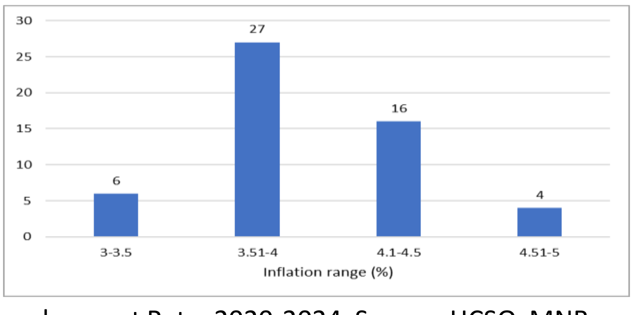


Fig. 1. A. The Phillips Curve for Hungary. Inflation: CPI, Unemployment Rate. 2020-2024. Source. HCSO, MNB, own Compilation

B. Frequency Distribution of Inflation Occurrences in 4 Ranges (2020-2024). Source: HCSO, MNB, own Compilation

Recently, in light of the Covid-19 crisis as well as the countercyclical fiscal and monetary policies, described in Ball's paper [32] enacted by governments of developed and developing countries, the Philips curve had once again come under scrutiny. If, of course, the notion that lower interest rates do indeed promote economic growth, and not raise unemployment, what is the trade-off point at which policy makers are confident to declare that the war on inflation had been won? This was the main question for the central bankers as soon as 2021 [33]. Both the curvature as well the strength of the relationship of unemployment and inflation have been central questions in case of Hungary as well.

In Figure 1, we plot the Hungarian Phillips curve using a scatter diagram, to which we added a logarithmic trend and determined the $R^2 = 0.1657$, which is considered good for real-world data. The dispersion of the observed values bears strong resemblance with the curve in [34].The models in FED working papers follow an inverse-L representation of the Phillips Curve like [35]. The arguments are: higher inflation causes increased production, as firms benefit from low real wages, the labour market is differentiated between tight and normal [35].

$$\pi_t = \begin{cases} k^{tight}\hat{\theta}_t + k_u^{tight}\widehat{u_t} + \beta E_t\pi_{t+1} \; labor\ shortage \\ k_w\widehat{w}_{t-1} + k\hat{\theta}_t + k_u\hat{u}_t + k_\beta E_t\pi_{t+1} \; normal\ conditions \end{cases}$$

Where $k$is the slope of the curve, labour market tightness is θ_t, which is the relationship between job vacancies and unemployed workers. If $\theta > 1$, there is a labor shortage, $k^{tight} > k$, becomes steeper than normal, and if it is less than 1, the conditions are considered "normal". $\widehat{w}_{t-1}$ is a state variable of wages, u_t are supply shocks. Lastly, it needs to be stated that from a monetarist school and from a QTM perspective, the equations defined above may be misleading. The numeraire that enables us to put a price on a commodity or service is money. None of the Phillips curve models incorporate changes in the money supply as a factor for rising prices that destabilizes natural rate of unemployment. No recent article published really questioned the direction of causation of the Philips curve. Does a reduction of inflation cause a rise in unemployment, or does a rise in unemployment cause a reduction in inflation?

Recent papers merge the Phillips curve with Real Business Cycle hypothesis (RBC). The goal is to provide a unified framework that detaches inflation expectations as well as explaining the great inflation of the 1970s and the inflation of the 2020s. Works that focus on these aspects include [36; 37; 38]. The accelerationist view is hard to justify from the perspective of Alvarez et al. [39], the contrary Boissay et al. [40] found due to the fact that firms have a much stronger pricing power than before, higher mark-ups may fuel inflation. Inflation is always and everywhere a monetary phenomenon said Milton Friedman. As long as the money created by commercial bank lending can be efficiently invested in successful productivity enhancing activities, the growth of the economy will inevitably outpace inflation. Maintaining growth is possible and it needn't be inflationary (Japanese economy from 1980-2000). From the point of view of the authors, the cause of inflation is the devaluation of currency, all else, are cost-push and demand-pull effects, they are:

4. Results and discussion

4.1. Experimental Design and the Derivation of the Phelps-Friedman Phillips Curve

The games described in the subsequent part of this paper have many individual players. To simulate the laboratory conditions in which the game should progress, we used machine learning algorithms to show the rational decisions of each participant. Note that when deriving the following equations, the simulation ran with virtual participants, who determined their price strategies via trial and error and not via any of the 4 rules. Consider a group of N participants who compose a market economy. Each participant assumes the role of a producer or a delegate of a company CEO. The producers make a profit by selecting an optimal price$p$. Following the experimental design as in Lambsdorff et al. [17], the goal for each player is to maximize the profit function in each round of the game given by

$$F(y,p) = yP - y(P-10) \qquad (1)$$

Where, yP is the total revenue from the production, $y(P-10)$ is the total cost of production$C$, and the unit cost c is determined by $P-1000$. The average of prices chosen by all participants in round n is denoted by P. This determines the costs, and it becomes known at the end of each round, when the producers announce the prices. The demand function for each player defined as

$$y = \omega - p \qquad (2)$$

The players act independently from each other, as this is a non-cooperative game. The lower the price p chosen, the higher the demand$y$. ω is an exogenous variable, which indicates the maximum possible demand at $p = 0$. There are 12 rounds in a game, representing the 12 months of a financial year. In the first 6 rounds (months)$\omega = 30$. In the rest of the rounds up until round 12, $\omega = 50$. The participant with the highest profit at the end of round 12 wins the game. In other words, the players must maximize the profit function given in equation (2). To achieve robust results, the game is simulated 10000 times with 300 participants, 400 and 500 participants as well. The results of the robustness tests are presented in Section 4.2.

To sum up, from the perspective of the producers, the steps to play are as follows:

Choose a price p that the player believes will maximize their profit

Calculate the demand y based on the chosen price

Determine the costs by plugging the chosen price in the cost function$c$. The average price P chosen by all participants will determine the production cost per unit, which is$P-10$.

Compute the profit by considering the revenue py and subtracting the total costs, determined by $y(P-10)$

Let's consider equation 1, we can express it through ω as

$$F(y,p) = (\omega - p)p - (\omega - p)(P-10) \qquad (3)$$

To maximize the profits, the first derivative is taken with respect to p and the new profit function is set to 0 to find the maximum.

$$\frac{\partial p}{\partial yp} F(y,p) = 0$$

$$\omega - 2p + P - 10 = 0 \qquad (4)$$

By rearranging the above, we end up with

$$p = \frac{\omega + P - 10}{2} \qquad (5)$$

This tells us that prices are strategic components. Each producer supplies the product to all the other ones, so each producer receives from all the other ones their production as an input for one's own production. Every player is simultaneously a price taker and a price giver. In such a case the inputs from the others are priced according to their price. If P is too large on average, one is forced to add on (to mark-up) on these costs by increasing one's own price. Consider the below illustration. If the price were fixed atP_t, corresponding to a quantity ofy_t, we would be facing the rectangle t bounded from the bottom by the coast line C and from the top by the fixed priceP_f. Assume that we had taken a low price, the rectangle f that was formed would be flat, conversely, if the price picked would have been too high,P_t, the rectangle would be too thin. An optimal choice to fill the total area under by the curve would be to pick a price somewhere in the middle $-P_o$, which would be the optimum price. An optimum total profit is therefore of a quadratic form. However, if we observe that the cost line C increases, then we are pushing upward the price, as well and the optimum profit must readjust.

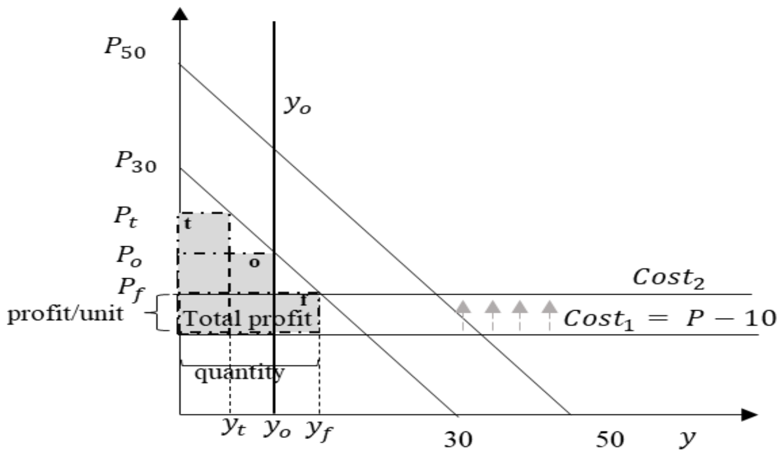


Fig. 2. Average Price set by the Group and Demand Relationship. Demonstration of the Long-Run Phillips Curve
Source: Own Compilation

Finding an equilibrium means finding a symmetry, i.e. all the market participants find a rational solution. Therefore, p chosen by one producer must equal to the price chosen by others$P$, that is, $p = P$ .Having recognized this, let's transform (4) as follows:

$$p = \omega - 10 \qquad (6)$$

Inserting (6) to the demand function, we get

$$y = 10 \qquad (7)$$

A. Rational expectations
A (Long-Run Phillips Curve)

B. Limited rationality
B (Short-Run Phillips Curve)

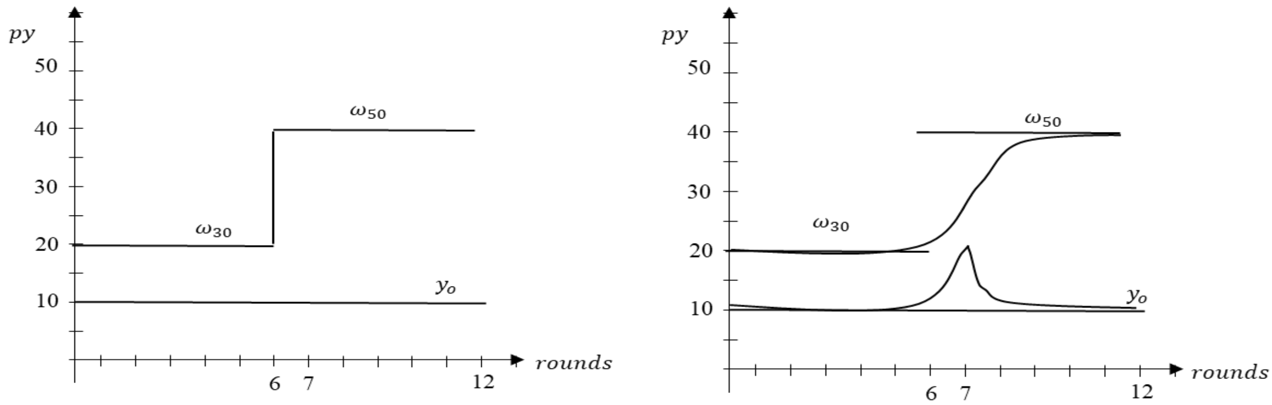


Fig. 3. Model of the Phillips Curve based on the Pricing Decisions of Participants
A - Rational Expectations, B - Limited Rationality
Source: Own Compilation

As we can see, in the long run demand doesn't drive up production, it only impacts prices (Figure 2). Rationality requires the participants to set prices such that the quantity is constant. The solution is therefore y_o – a vertical line for production. That is the long run Phillips curve. When one has a first look at the game, one is tempted to believe that the quantity produced will increase withω. And the reason is because an increase in ω translates to higher prices$p$, but then translates to higher costs, the higher costs again translate to higher prices, and this turns into a spiral. Whatever level ω is chosen, it would only correspond to one point on the curve. This is the long run Phillips Curve, in other words in the long run demand doesn't drive up production, and it only impacts prices. However, this is not how the game is immediately perceived. Figure 3 illustrates the expectations of the price behaviour given a change in ω according to rational expectations theory, and the limited rationality theory to which the authors of this paper subscribe. The results of the simulation support B.

We expect that the quantity should always equal 10 and prices should equal 20, calculated as (30-10). As ω increases to 50, we expect an immediate jump in the price level to the higher level of 40 (50-10). The simulations in Figure 4 show some heterogeneity in the first stage. But notice how the prices didn't make a discrete jump to 40, but rather there was a mild increase followed by a convergence to 40. What happens when the price isn't increased fully is that demand is at a disequilibrium. When ω is at 50, p is only at 30, the demand is not at the equilibrium level of 10, but rather above (Figure 3B) – this is the short-run Phillips curve.

Figure 4 shows the results of the simulation given the following input variables: Participants: $30, \omega_1\ (rounds\ 1-6) = 30, \omega_2 = 50$. Number of simulations: 10,000.

Let's discuss the reasons proposed by theory. How exactly should we interpret the results in Figure 4, and why is it so similar to Figure 3B? Consider (5). Participants don't perfectly apply that function in practice and the corresponding logic for symmetry, rather they form expectations with respect to P . In other words, they don't infer prices from the model, but rather, they form expectations:

$$p = \frac{\omega + E(P) - 10}{2} \qquad (8)$$

Market participants form expectations with a sense of inertia, they expect unit costs (average price) to be equal to the observations in the previous round. That is, $E(P) = P_{-1}$. This is sluggishness or inertia.

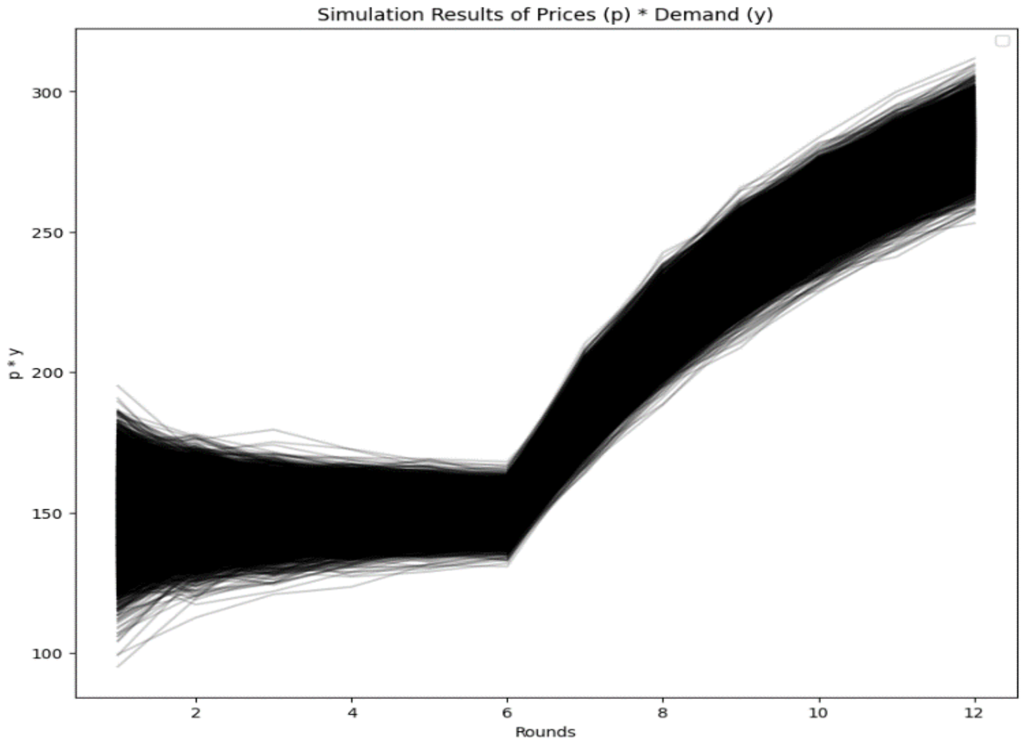


Fig. 4. Results of the Simulation: Realized Profit of Participants in Rounds 1-12. N = 30 Participants, 10,000 Simulations
Source: Own Compilation. Source-Code: Appendix A

Plugging in the numbers in (8), in round 6, the average price was 20, so 50 + 20 -10 is 60. Divided by 2 is 30. That is exactly what was found. Table 2 summarizes the chosen price based on the simulation for a representative sample of simulations number 1, 60, and 100 for rounds 6 and 12.

Table 2
Sample Simulations 1, 60,100, Rounds: 6, 12

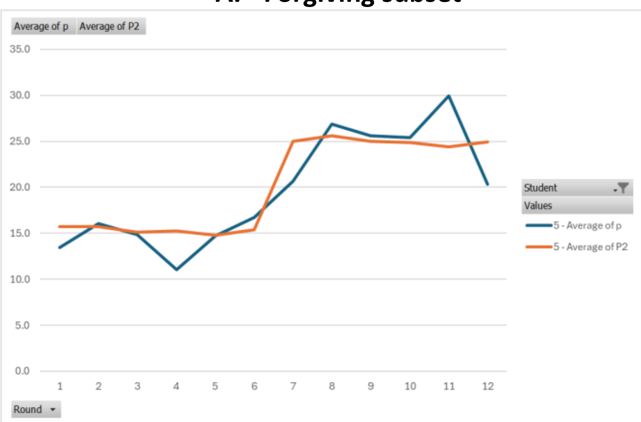
Sim 60, Round 6, Omega 30				Sim 1, Round 6, Omega 30				Sim 100, Round 6, Omega 30			
Player	p	y	P	Player	p	y	P	Player	p	y	P
1	3.23	26.77	14.13	1	14.70	15.30	17.32	1	23.06	6.94	14.31
2	11.45	18.55	14.13	2	16.94	13.06	17.32	2	14.29	15.71	14.31
3	6.03	23.97	14.13	3	28.26	1.74	17.32	3	29.82	0.18	14.31
4	21.16	8.84	14.13	4	26.18	3.82	17.32	4	8.64	21.36	14.31
26	6.13	23.87	14.13	26	11.17	18.83	17.32	26	9.77	20.23	14.31
27	23.59	6.41	14.13	27	12.89	17.11	17.32	27	9.84	20.16	14.31
28	20.80	9.20	14.13	28	20.93	9.07	17.32	28	29.47	0.53	14.31
29	0.01	29.99	14.13	29	3.86	26.14	17.32	29	0.01	29.99	14.31
30	25.19	4.81	14.13	30	22.29	7.71	17.32	30	3.34	26.66	14.31
1	39.80	10.20	21.78	1	49.16	0.84	22.40	1	13.84	36.16	22.11
2	17.08	32.92	21.78	2	4.52	45.48	22.40	2	25.08	24.92	22.11
3	33.95	16.05	21.78	3	36.04	13.96	22.40	3	47.12	2.88	22.11
4	44.62	5.38	21.78	4	16.66	33.34	22.40	4	1.88	48.12	22.11
26	1.79	48.21	21.78	26	5.37	44.63	22.40	26	35.16	14.84	22.11
27	23.64	26.36	21.78	27	49.83	0.17	22.40	27	26.66	23.34	22.11
28	13.87	36.13	21.78	28	16.81	33.19	22.40	28	23.74	26.26	22.11
29	3.72	46.28	21.78	29	0.42	49.58	22.40	29	22.21	27.79	22.11
30	6.43	43.57	21.78	30	14.62	35.38	22.40	30	16.11	33.89	22.11

Source: Own Compilation. Source-Code: Appendix B

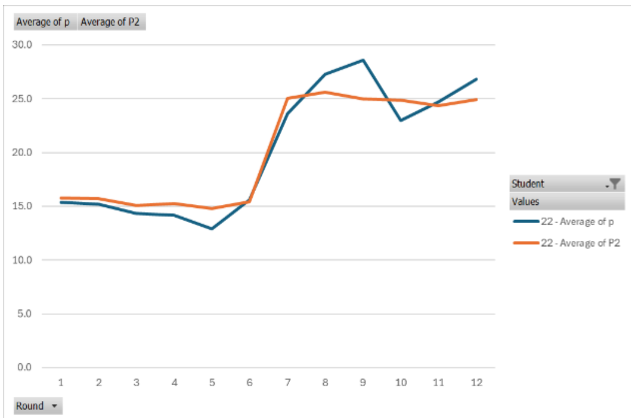
4.2. Explanation of the Results of the Simulation

In the 1970s Lucas and Phelps, advanced the idea that price takers don't know how the unit costs are determined, therefore equation (8) is a black box to them. They don't know where the unit costs come from (the prices of all the other market participants), because they are not directly observable, the participants take them as given, and according to the simplest explanation, they look at the previous price and integrate it into the current one. This is not a relevant explanation for the behaviour in our game since there is no limited information in the game. The participants know how the average price P is formed. They would have the capacity to infer the equilibrium, but they don't do it. Therefore, another explanation is needed. Limited rationality: Participants don't reason all the way through to come up with the equilibrium, instead they form higher order beliefs and infinite levels of reasoning. This is false, as well, because as evidence shows they carry out at most 1 level of reasoning. People don't do this. We performed 10,000 simulations on 300 virtual participants using one of the 4 cooperating rules. None of them were found to deliver superior results than if the participants were figuring out the prices via trial-and-error and maximizing their profit function independently without cooperation.

A. Forgiving subset



B. Tit-for-Tat subset



C. Competitive subset **D. Random subset**

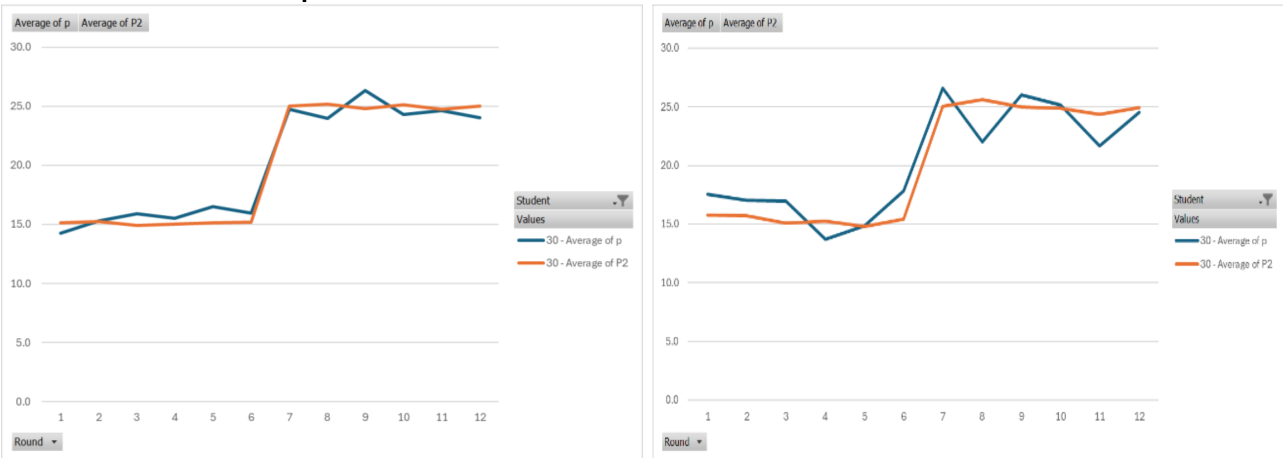


Fig. 5. Average Price set by Players (Orange) Price set by Player's following: A - Forgiving Rule, B - Tit-for-Tat Rule, C- Competitive Rule, and D- Random Rule. Note that Random is not equivalent to the Trial-and-Error Method used in the Derivation of the Phillips Curve.
Source: Own Compilation. Source-code: Appendix B.

4.3. An Extension of the Model

Let's extend the model and change ω in the sense that instead of a discrete jump (variable) from round 6 to round 7, let's assume that ω develops continuously over time. In such case, it is useful to form a slightly more complex function for the formation of expectations:

$$\frac{E(P)}{P_{-1}} = \frac{P_{-1}}{P_{-2}} \qquad (9)$$

Or multiplying both sides by P_{-1} we get

$$E(P) = \frac{P_{-1}^2}{P_{-2}} \qquad (10)$$

We can now substitute (10) into the rearranged equation (8) as follows:

$$p = \omega + \frac{P_{-1}^2}{P_{-2}} - 10 - p \quad (11)$$

Now let's divide (11) by P_{-1} such that:

$$\frac{p}{P_{-1}} = \frac{\omega - 10 - p}{P_{-1}} + \frac{P_1}{P_{-2}} \quad (12)$$

Instantly we recognize inflation π in the term $\frac{p}{P_{-1}}$. Although the true definition of inflation π is $\frac{p-P_1}{P_{-1}}$, therefore, in (12) we must subtract $\frac{p}{P_{-1}}$ from both sides, for the sake of simplicity, let's contend with the simpler $\frac{p}{P_{-1}}$ definition. One can also recognize that equation (1) is contained in (12), so it is easy to replace $\omega - p$ with the demand function y. The natural rate of output further in this paper denominated as $\bar{y}$ is the long-run equilibrium of output in our model, which is 10. So, it will be useful to generalize and use $\bar{y}$ instead of 10.

$$\frac{p}{P_{-1}} = \frac{y - \bar{y}}{P_{-1}} + \frac{P_1}{P_{-2}} \quad (13)$$

4.4. The Standard Phillips Curve

A standard variant of the Phillips curve (as in Friedman (1968) and Phelps (1968)) is as follows:

$$\pi = \pi_{-1} + \gamma\tilde{y} \quad (14)$$

Where π is inflation at time$t$. π_{-1} is inflation 1 period ago, γ is the inverse of the price elasticity of demand, $\tilde{y}$ is the output gap, which is a proxy for the unemployment gap, and is calculated as $\tilde{y} = \frac{y-\bar{y}}{\bar{y}}$, where y is the aggregate demand, and $\bar{y}$ is the potential production, which is the level of production in the economy if all the resources, such as land, labor and capital are adequately utilized, and there is no excess capacity left. When aggregate demand is equal to potential production, inflation will stay constant and there will be no upwards pressure on inflation. If the aggregate demand is in excess of the potential production, then the excess will push inflation upwards by the γ parameter. Current literature works with multivariate VAR versions of the Phillips curve presented in (14), like Benigno and co-authors [35]. A method of deriving this equation is built on limited information and limited rationality. Another variant is built upon rigidity i.e that the producer isn't always able to adjust prices, sometimes they need to leave prices unchanged [7]. That is a different type of Phillips curve known as the New Keynesian Phillips curve. Most ECB and BIS publications follow Jordi Gali's [41] New Keynesian hybrid Phillips Curve framework for determining the inflation-output tradeoff.

4.5. The New Keynesian Phillips Curve. Game 2

The New Keynesian Phillips Curve is different from the classical one insofar as it features the relationship between forward inflation expectations and the output gap:

$$\pi_t = \pi_{t+1} + \gamma\tilde{y} \qquad (15)$$

Compared to (14) one noticeable difference is that the equation contains the forward-looking inflation variable π_{t+1} instead of the backward-looking one. γ is some parameter and $\tilde{y}$ is the output gap. To better understand this equation, let's consider a different game. There are n games played. In each game, the participant must get their price as close as possible to a target value over 12 rounds (again representing the months of a financial year). Instead of direct price submissions, in this game, players submit price proposals, that are accepted with a probability$\alpha = \frac{1}{3}$, and not accepted with a probability$1 - \alpha = \frac{2}{3}$, and the price of the previous round remains in effect. The loss function needed to be minimized is

$$F(x, z) = (x_t - z_t)^2$$

In the beginning the target value is 100. The player will start with a price of 100, identical to the target value. However, starting in round 7, the target value suddenly drops from 100 to 60. So, when the player submits a price proposalx_t, then this will be implemented with a probabilityα, and with a probability $1 - \alpha$ it will be rejected, and the old price remains in effect. By repetitions, the machine learning algorithm can try again through trial and error and learn how to play the game. The probabilities of price acceptance introduce rigidity into the system. In other words, as a producer the player can't directly determine their price, there are some rigidities, in the above case, for example, there is a chance that their price will simply not be realized. In another case there might be costs to adjusting the price. The players may sometimes prefer not to adjust the price, as the adjustment costs are too high (buying new machinery, retraining staff, creating a new pricelist). There can be many causes of rigidities, however this is a particular case which was advanced by [7]. The goal of the players is to by choice of x_t the sum of all future square deviations.

$$min_{x_t} \sum_{j=0}^{\infty}(1-\alpha)^j\left(x_t - \mathrm{z_{t+j}}\right)^2 \qquad (16)$$

The price proposal is implemented with probability$\alpha = \frac{1}{3}$. Thereby, for any round t the price P_t may be determined. It will depend on a proposal x_t the price equation is:

$$P_t = \alpha x_t + (1-\alpha)P_{t-1} \qquad (17)$$

Equation (17) indicates a certain degree of inertia resulting in the Phillips curve itself having some inertia. Nevertheless, the New Keynesian Phillips curve is forward looking. This is because the challenge of the participants is that in round 7, the price proposal of 60 must be implemented, because nothing else will make sense. The player's optimal pricing strategy would be to lower the price in round 7, as keeping it above that level will result in penalties through the loss function. But because of the nature of rigidities (a genuine uncertainty present, where the price may not be implemented, and the loss will have to be realized) the player might already in round 6 observe that a lower price proposal makes sense. The reason for that is that if the price in round 6 is implemented, the player on one hand will definitely suffer from the loss function in the round but will possibly avoid even larger losses in subsequent rounds, were the price not implemented for a number of rounds after round 6. The quadratic loss function implies that small deviations are not as impactful as large deviations, since they hit a hypothetical account in a quadratic form. From the perspective of the players, what is surely not an optimal price proposal setting strategy, is to set a price proposal in round 6 of 100, because in that case losses in round 6 are avoided completely, but if the price proposal of 60 in round 7 is not realized, then the player would suffer a large loss, had they not lowered their price beforehand. The price development therefore should be smoothed.

4.6. How to Infer the Optimum Path of Price Proposals?

Let's assume that in round$t$, the price proposal x_t is realized, and we take this as a starting point for finding the optimum value. We disregard the possibility of the price proposal being rejected, for if it is rejected, it would be completely irrelevant to proceed further. If x_t is realized, there is a possibility that it will not be updated until$t+j$. That is, for j rounds the price may not be updated. That probability is expressed as $t+j=(1-\alpha)^j$ and we can illustrate it with an exponential function that approaches zero, but never quite reaches it. The y-axis are the probabilities, and x-axis represents the rounds.

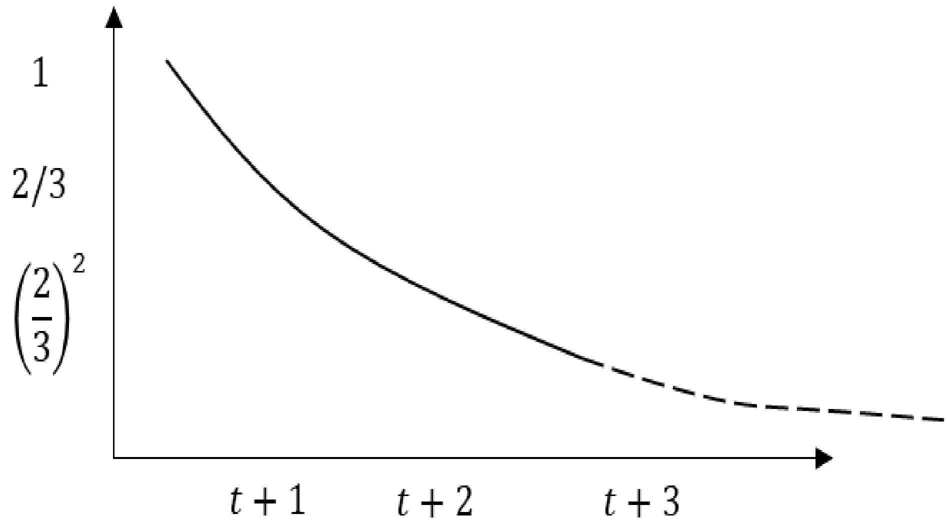


Fig. 6. Illustration of the Probability Curve of the Price Proposals being accepted for t+j Rounds.
Source: Own Compilation

In round 6 when choosing an appropriate price proposal x_t one must compare two costs. On one hand, there is a certain and immediate cost in the current round if the proposal is implemented, and the strategy of smoothly approaching the next target price is followed. We are looking for a cost minimum of (16). In the game it only runs until round 12, however, using ∞ allows for some applications for infinite rows of numbers. To minimize (16), we need to take the first derivative and equate it to 0.

$$\frac{\partial L}{\partial(x_t)} min_{x_t} \sum_{j=0}^{\infty}(1-\alpha)^j\left(x_t-\mathrm{z_{t+j}}\right)^2 = 0 \quad (18)$$

$$\sum_{j=0}^{\infty}(1-\alpha)^j\left(2x_t-2\mathrm{z_{t+j}}\right)=0 \quad (19)$$

Simplified to

$$\sum_{j=0}^{\infty}(1-\alpha)^j x_t = \sum_{j=0}^{\infty}(1-\alpha)^j z_{t+j} \quad (20)$$

We know that x_t is a constant, therefore, we can come to a simple formula on the left-hand side of (20) by utilizing the common summation rule $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$. By substituting the rule on the left-hand side and expressing the relationship in terms of $t+1$, we get

$$\frac{1}{\alpha} x_{t+1} = \sum_{j=0}^{\infty} (1-\alpha)^j z_{t+1+j} \qquad (21)$$

The right-hand side can be rewritten by shifting the running index one round ahead. So, j starts with 1. At the same time, each time we refer to the running index, 1 must be subtracted from the power of $(1-\alpha)$ and the index of z_t.

$$x_{t+1} = \alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} z_{t+j} \qquad (22)$$

Let's consider (21) when $j = 0$

$$x_t = \alpha \sum_{j=0}^{\infty} (1-\alpha)^j z_{t+j} \qquad (23)$$

The sum on the right-hand side of (23) can be broken up into two parts:

$$x_t = \alpha \sum_{j=0} (1-\alpha)^j z_{t+j} + \alpha \sum_{j=1}^{\infty} (1-\alpha)^j z_{t+j} \qquad (24)$$

The first part is where $j = 0$ and the second part is where is from $j = 1$ to ∞ . Substituting 0 for j in the first part, we will get αz_t. The second part of the equation is almost identical to the right-hand side of (22). To make it fully identical, we must multiply x_{t+1} by $(1-\alpha)$.

$$x_t = \alpha z_t + (1-\alpha) x_{t+1} \qquad (25)$$

This solution allows us to understand how one should determine the optimum. The optimum path of the price proposal denoted as $x_{optimum}$ will take the shape of a curve. In the Newy Keynesian Phillips curve history plays no role. However, the NKPC is still not ready yet. We need to infer it from (25). First, let's subtract the price level P_t from both sides of the equation, while linking the price level to the target.

$$x_t - P_t = (1-\alpha)(x_{t+1} - P_{t+1}) + \alpha(z_t - P_t) + (1-\alpha)(P_{t+1} - P_t) \qquad (26)$$

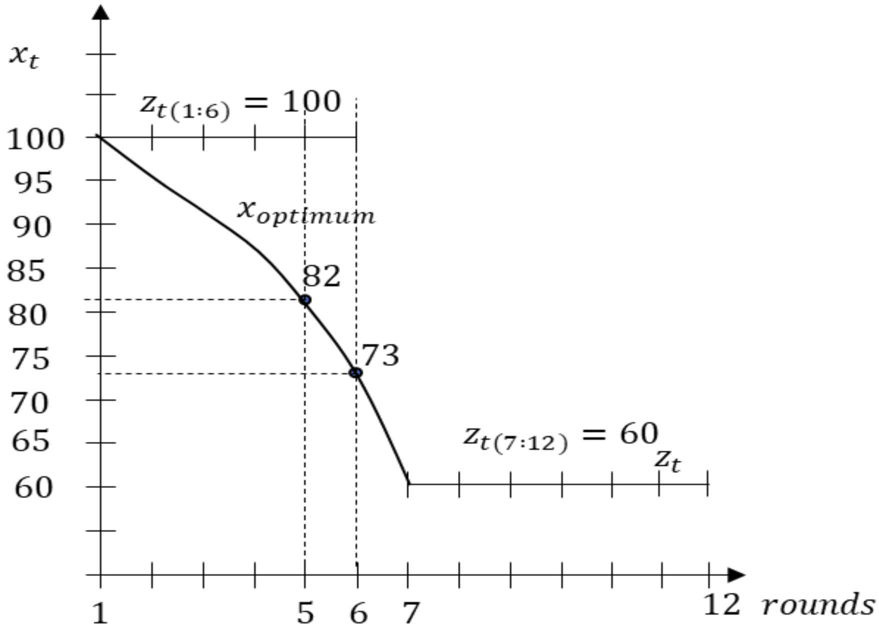


Fig. 7. Illustration of the Optimal Price Strategy of Participants in Game 2
Source: Own Compilation

Is a function that refers to prices, and we know them insofar as they are observable. However, both (25) and (26) are functions that are related to price proposals, which are unobservable. So, we must somehow transform the equations into the observable realm. To do this we turn to (17) and subtract the price levelP_t.

$$(1-\alpha)(P_t - P_{t-1}) = \alpha(x_t - P_t) \qquad (27)$$

If we divide byα, we now see that $x_t - P_t$ (equation (26)) can be expressed purely in prices. So, inserting (27) in (26) we get

$$\frac{1-\alpha}{\alpha}(P_t - P_{t-1}) = \frac{(1-\alpha)^2}{\alpha}(P_{t+1} - P_t) + (1-\alpha)(P_{t+1} - P_t) + \alpha(z_t - P_t) \qquad (28)$$

Multiplying (28) on both sides by $\frac{\alpha}{1-\alpha}$ will simplify the equation to

$$P_t - P_{t-1} = (P_{t+1} - P_t) + \frac{\alpha^2}{1-\alpha}(z_t - P_t) \qquad (29)$$

Dividing both the left and right sides by P_t, we get current inflation π_t on the left side, π_{t+1} future inflation on the other side added to the output gap. Considerz_t, which the target price is. That could be interpreted as the one price that brings demand in line with potential production. That is reasonable, when the proposed price is at the target level, in other words the production is in balance with the potential production. When the current price deviates from the target value, which means demand doesn't match the production, which is the output gap. We can infer for round 7, for example: we have a price target of 60, but the price, on average, will be above, because we will have set price proposals that were above 60 previously. And on average the price proposal of 60 will not be realized. Therefore, when the price target is lower than the observable price, the whole $\frac{\alpha^2}{1-\alpha}(z_t - P_t)$ term becomes negative. That corresponds to a negative output gap – a recession. In a recession, current inflation is pressed downward.

4.7. Robustness Tests of the Simulations

We performed robustness checks on the obtained 120,000 observations. The goal of the descriptive statistics in Table 3 are to measure the Root Mean Square error of all of the 30 participants within the 12 rounds. It can be noted that as the rounds progress the RMSE stabilizes, indicating the progression of the Machine learning algorithm through the stages of the trial-and-error method. The MAE stabilized as well. To investigate whether the number of simulations has been picked optimally in this study, we checked the robustness of the results by varying the number of simulations, and participants. The number of simulations were varied between 5,000 to 15,000 and the number of participants from 200 to 400. If the differences in means converge to zero, then no advantage is gained by the variation. In Figure 8 we plotted the changes in RMSE and MAE as well as the mean relating to the revenues py earned by the participants.

Table 3
Descriptive Statistics of the Average Revenue realized by Participants, calculated as P*y, for 30 Students, 10,000 Simulations. Mean, Root-Mean-Squared-Error, Mean-Average-Error

Round	Mean (P*y)	RMSE	MAE
1	150.07	12.29	9.82
2	150.02	8.64	6.92
3	149.99	7.08	5.65
4	150.04	6.11	4.86
5	150.03	5.48	4.38
6	150.04	4.98	3.98
7	188.08	6.51	5.22
8	216.67	7.12	5.70
9	238.91	7.36	5.87
10	256.67	7.42	5.91
11	271.21	7.43	5.91
12	283.34	7.40	5.90

Source: Own Compilation. Source-Code: Appendix C

A. Difference between 5.000 simulations and 10,000 simulations

B. Difference between 5.000 simulations and 15,000 simulations

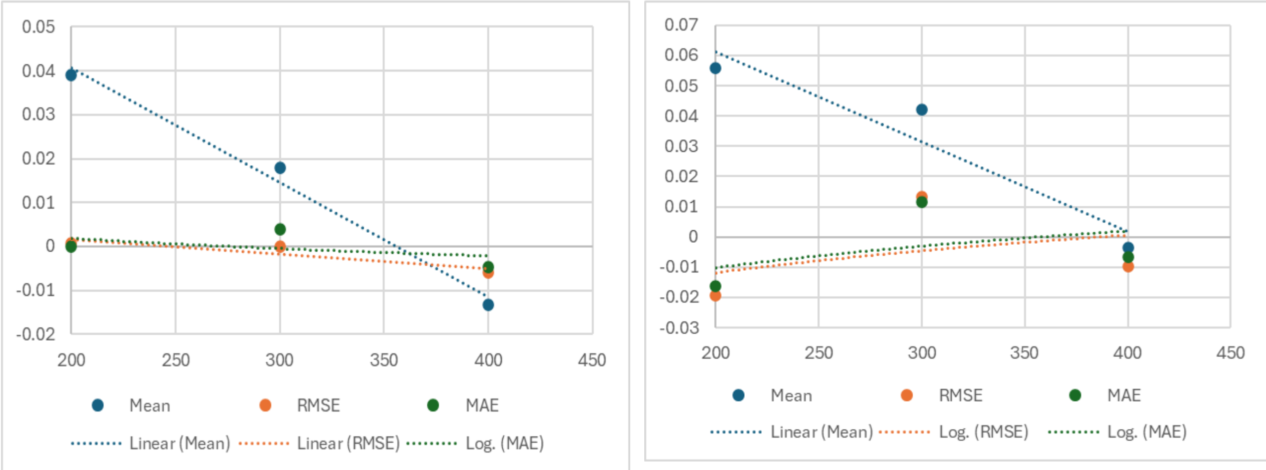


Fig. 8. Does increasing the Number of Simulations Lower RMSE, MAE?
Source: Own Compilation
Source-Code: Appendix C

No significant advantage was found to be gained by raising the number of simulations based on a convergence of the Mean metric to zero. All of the metrics show convergence with the rise of the number of participants. In our research MAE indicates how well the simulated values fit the expected outcomes. A change in the MAE throughout the rounds, as displayed in Figure 9. Shows that as the rounds progress, the MAE first drops to round 6, indicating a consensus between participants, and then it rises in a logarithmic shape. The variability increases in round 7, when a structural shock in terms of the external demand variable ω is introduced. Both the ANOVA and Kruskal-Wallis test results indicate significant differences between rounds at 10% significance.

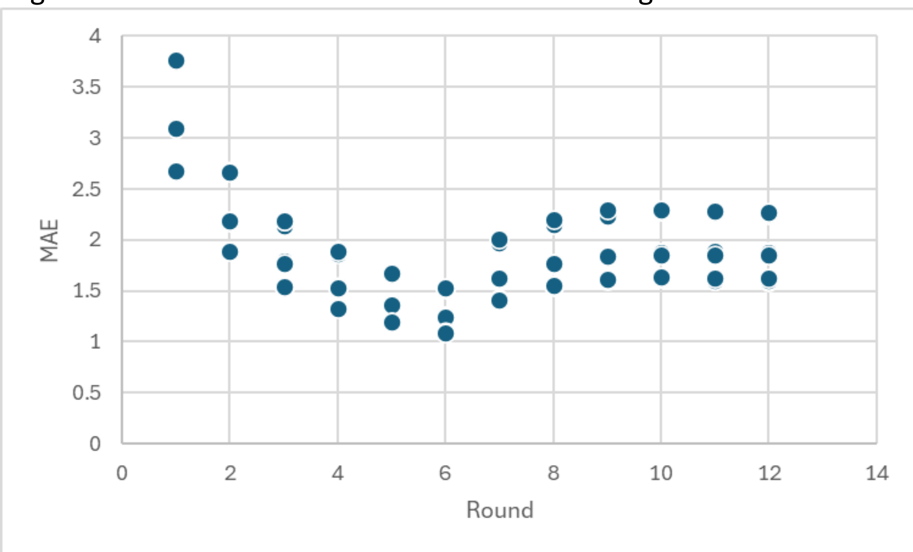


Fig. 9. Average MAE per Round
Source: Own Compilation
Source-Code: Appendix C

5. Conclusion

After nearly 70 years, the controversy sparked by William Phillips' original introduction of the Phillips Curve into econometrics remains unresolved. The debate surrounding the direction of causality, correlation, and the dynamic relationship between unemployment and inflation continues to generate considerable interest. It remains a fertile research area, inspiring new econometric methodologies and policy proposals. In this paper, we approach the Phillips Curve problem from the perspective of game theory, grounded in the rational expectations framework proposed by Friedman and Phelps in 1968. Our study introduces a hypothetical scenario devoid of a central bank, supply fluctuations, monetary effects, or capital considerations, and yet, we observe the emergence of the Phillips Curve through our simulation’s results. In Game 1, we designed the participants (virtual agents) in this simplified economy to maximize their profit functions based on demand. The players were equipped with advance knowledge that demand would fluctuate in round 7. However, instead of a sharp adjustment in response to this expected shift, the results revealed that participants adjusted their strategies gradually, reflecting backward-looking expectations. This behaviour illustrates a reluctance to make sudden price changes even when armed with clear information about future fluctuations, revealing a non-rational expectation element that contrasts with traditional economic models. In Game 2, we introduced the Calvo economy, a model where firms adjust prices with some inertia due to adjustment costs, which aligns with the costliness of changing prices frequently. This scenario allowed us to derive the New Keynesian Phillips Curve, characterized by forward-looking expectations. Essentially, Game 2 presents a reverse of the dynamics observed in Game 1. Players, in this case, adjusted their pricing strategies based on a quadratic loss function, minimizing this function as they set prices. Central banks today use similar quadratic loss functions when constructing New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models, underscoring the relevance of our simulation to real-world economic policymaking. It is crucial to acknowledge that deriving macroeconomic policy solely from the outcomes of this simulation would be misguided. Specifically, the results do not suggest that central banks should abandon their dual mandates of full employment and price stability in favour of market self-regulation. Instead, this simulation illustrates that, both in practice and in theory, the price-setting mechanisms among our virtual players are subject to the law of limited rationality. Participants following a trial-and-error approach do not base their decisions on dynamic stochastic general equilibrium models predicting rational expectations. Rather, they compare current price movements with past outcomes, adjusting strategies to maximize profit within the limitations of their rationality. While the DSGE model inferred from their price-setting strategies offers a good approximation of their behaviour, it remains, as is the case with most models, an approximation at best and a misleading representation at worst. Real economic conditions are more complex, and the rigid assumptions underlying the DSGE framework may fail to capture the full spectrum of market behaviours. Another important consideration is the extent to which the participants in this study were programmed to anticipate changes in demand. In this simulation, participants knew in advance about demand shifts, meaning anticipation played a minimal role. Had the program introduced uncertainty, such as assigning a random probability for demand to either drop by 50% or rise by 200% in round 7, the results might have differed. In particular, cooperative pricing rules could have led to higher revenues. However, we predict that even in such scenarios, the adjustment process would remain smooth, as observed in Game 2. This aspect of the study, though highly speculative, warrants further research, as it was beyond the scope of this particular investigation. In reality, predicting demand is far more difficult. Even the most advanced Vector Autoregressive (VAR) models struggle to account for "black swan" events unpredictable and highly impactful occurrences and are prone to overfitting. As of this writing, we argue that the most reliable method of estimating inflation is through a combination of multivariate analysis and the New Keynesian Phillips Curve (NKPC). Although the findings of this simulation offer valuable insights, they should be interpreted cautiously, particularly in regard to the Phillips Curve's practical implications for real-world macroeconomic policy.

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