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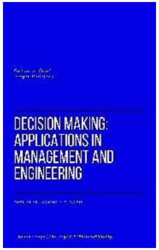

# Multi-Criterion Support for the Decision Problem Solving in the Food Packaging Process

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**ABSTRACT**

The paper highlights the problem of the two-stage procedure for optimizing food packaging where the first stage involves selecting the optimal packaging structure and the second stage allows for the optimal selection of parameters of the packaging machine taking into account two criteria: the efficiency of the packaging process and the oxygen content in the packaging. The use of the modified Baas and Kwakernaak method in an industrial experiment allowed for the determination of the optimal packaging configuration assuming there are two types of criteria: deterministic ones focusing on the unit cost of packaging and tightness of the packaging and fuzzy ones focusing on appearance, smell and taste of the packaged product. The optimal packaging variant is variant a7 with the highest weighted average rating value equal to 0,7113. As part of parametric optimization, based on the obtained experimental results, the analyzed optimization criteria are presented in the form of regression equations and then these equations are subjected to statistical analysis. The form of the substitute criterion is formulated for the resulting single-criterion optimization problem. To deter-mine the set of Pareto optimal variants, the weight method is used, changing weight values every 0.05. Finally, the best variant is selected due to two opposing criteria using the distance function method implementing the Euclidean metric. The decision variables of the optimal variant also constitute the optimal parameters of the packaging machine due to the adopted optimization criteria. The optimal variant of the packaging process is variant 9 for which the value of the distance function di[f(x)] reaches the lowest value, i.e. 0.5533.

## 1. Introduction

The competitiveness of many companies is strictly dependent on the accuracy of decisions accompanying information and material flows within the supply chain [1; 2]. Therefore, as part of the broadly understood distribution organization in the food industry, one of the practices is to extend

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the shelf life of the finished product. This approach creates opportunities for planning deliveries, taking into account both the expectations of recipients and the guidelines determining the planning and scheduling of production [3-5]. In recent years, a dynamic increase in demand for convenience food has been noticed – fresh food, without preservatives, with high shelf life, ready to eat after short processing is required more and more. The answer to these consumer requirements is food packaged in vacuum conditions or in a modified atmosphere [3]. In conditions of increasing competition on the market of food producers, the quality of decisions made in the area of production and distribution of finished products becomes particularly important [6]. According to the authors, an important measure to meet the above expectations is the proposal of a quantitative method sup-porting decision-making processes.

Food is an important material for survival. The increasing world population, urbanization, and globalization are responsible for more food. This has increased challenges in food storage and safety [7; 8]. Therefore, it is necessary to preserve food by suitable packaging materials [10]. The packaging materials are useful for giving longer life to the food and improving quality during transportation, storage and distribution. Innovations and developments in food packaging, have become very important in the food industry [9].

The essence of vacuum packaging comes down to the elimination of the primary atmosphere (air) from the package, and then its tight closure by welding the elements that make up the pack-age, e.g. the lower foil with the upper foil. A more advanced packaging method is mixed gas pack-aging, also known as Modified Atmosphere Packaging (MAP). In this process, unlike vacuum pack-aging, the package is additionally completely or partially filled with a mixture of protective gases [10]. This creates an alternative to the methods used so far to extend the shelf life of food products using chemical preservatives [11]. The use of properly selected materials, taking into account the individual requirements of each type of food, while maintaining the initial atmosphere inside the package, can also affect the quality of packaged food products [12]. In the process of packaging food products, in particular meat products, they form the basic group of protective gases [13]:

- Nitrogen (N2),
- Carbon dioxide (CO2),
- Mixtures consisting of nitrogen and carbon dioxide.

The most common packaging materials used in the food packaging process include multilayer films (laminates). In the simplest form, multilayer films consist of the so-called carrier layer and a layer enabling welding. The carrier (outer) layer protects the packaged product against mechanical damage and against harmful chemical factors, and determines its stiffness, durability and printing possibilities. The sealing layer (inner) serves as a barrier to water vapour and as a medium to close the package [11].

Innovations in food packaging systems already help to meet the evolving needs of the market, such as consumer preference for “healthy” and high-quality food products and reduction of the negative environmental impacts of food packaging [14]. Emerging concepts of active and intelligent packaging technologies provide numerous innovative solutions for prolonging shelf-life and improving the quality and safety of food products [15].

The variety of means and methods of packaging food products, differing in labour intensity and cost, ensuring different properties of the packed product, with a shorter or longer shelf life, forces the evaluation and selection of the most rational configuration of the packaging film and the gas mixture [4].

There are many publications in which issues related to algorithms and quantitative methods supporting decision-making processes in the selection of the optimal option in terms of the following criteria are addressed:

- point [16; 17],
- fuzzy [18; 19],
- deterministic [20; 21],
- and deterministic, probabilistic and fuzzy [22; 23].

Currently, there are no studies on two-stage optimization, where the first step involves selecting the optimal packaging that meets the deterministic and fuzzy criteria, and the second step allows the selection of optimal operating parameters of the packaging device.

The goal of the work is the proposal to optimize the food packaging process based on the procedure consisting of two stages (Figure 1):

- structural optimization consisting in the practical application of the modified Baas and Kwakernaak method [23; 24] to select the optimal configuration (combination) of the packaging film and the medium inside the package (protective gas mixture) using deterministic and fuzzy criteria;
- parametric optimization, presented for example in De Marchi et al. [25], the purpose of which is to determine the optimal parameters of the packaging machine [26] through the implementation of active experiments allowing to find regression equations (objective functions), generating a set of Pareto optimal solutions [27] due to two objective functions and determining the best solution using the distance function method [28].

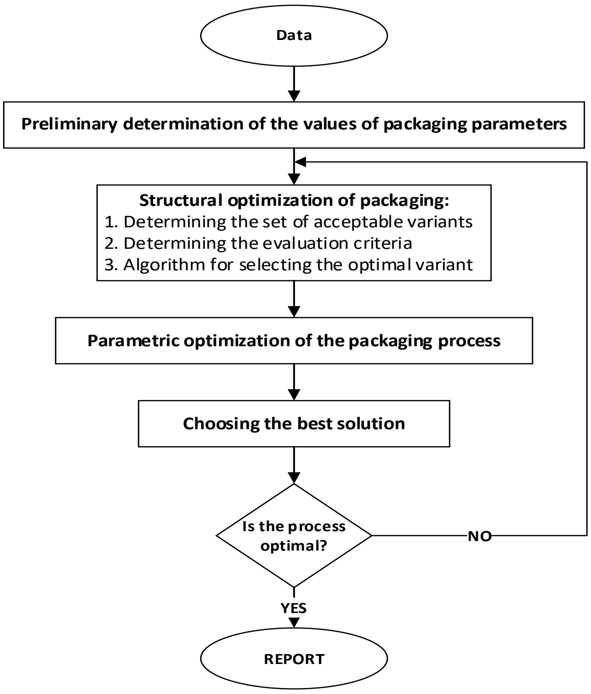

**Fig. 1.** Block diagram of multi-criteria structural and parametric optimization of the packaging process
**Source:** own study

The work consists of an introduction, structural optimization (packaging configuration), primary optimization (packing process), practical experiment and conclusions.

## 2. Algorithm for Choosing the Optimal Package Configuration Using Deterministic and Fuzzy Criteria

The input data in the algorithm for selecting the optimal option using deterministic and fuzzy criteria are taken from the following papers [23; 29]:

- The number of packaging options n,
- The number of experts appointed to assess individual options p,
- The number of deterministic and fuzzy criteria m.

Let A be the set of packaging options (1):

$$A = \{a_1, a_2, \dots, a_n\} \tag{1}$$

And K(dr) be a set of deterministic and fuzzy criteria against which individual packaging options are assessed.

Deterministic criteria refer to measurable quantities, such as the tightness of the packaging and the cost of packaging. Deterministic evaluations of variants are most often determined in various dimensions depending on the criterion and the adopted value scale. The values of these assessments should be transformed into the range <0.1>. Deterministic evaluations are treated as a special case of the fuzzy approach and are most often modelled with a membership function.

Fuzzy criteria very often concern packaging parameters and, consequently, the quality of the packaged product assessed through the prism of organoleptic features, such as external appearance, smell, taste or structure image. They constitute a group of subjective and vague criteria. The graphical interpretation of such criteria are triangular membership functions showing fuzzy assessments of individual variants.

In the first stage of the implementation of the algorithm, appointed experts perform a score evaluation of individual criteria using the Saaty method [30]. According to this method, the criteria are evaluated in pairs, each pair of criteria (ks,kt) is assigned an integer from the predetermined range <0,g>, where g is most often assumed to be the numbers 3, 5, 7, 9. These numbers determine preferences of the ks criterion in relation to the kt criterion. In this way, each of the experts builds criteria importance matrices (2):

$$\mathbf{B} = \left[b_{ij}\right], \text{ where: i = 1,...,m, \ \ j = 1,...,m.} \tag{2}$$

Then, on the basis of these matrices, triangular membership functions of fuzzy evaluations of the relative importance of the criteria µVj(vj) are created (Figure 2) where the values vj(min), vj(max) and vj(mod) are calculated from the following formulas (3):

$$\begin{aligned} v_{j(\min)} &= \min \boldsymbol{\lambda}_j(e) \\ v_{j(\max)} &= \max \boldsymbol{\lambda}_j(e) \\ v_{j(\mathrm{mod})} &= \frac{1}{p}\sum_{e=1}^{p} \boldsymbol{\lambda}_j(e) \end{aligned} \tag{3}$$

Where in (4):

$$\boldsymbol{\lambda}_j(e) = \frac{\lambda_j(e)}{\max \lambda_j(e)} \tag{4}$$

And λj(e), j = 1,...,m, is the eigenvector corresponding to the largest eigenvalue of the Saaty matrix of the criteria validity.

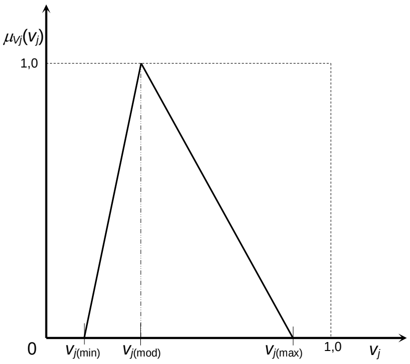

**Fig. 2.** Affiliation function of fuzzy evaluations of relative importance of criteria
**Source:** own study

In the second stage of the algorithm implementation, a group of experts proceeds to evaluate the packaging options in the light of the adopted criteria. In the case of assessing options against deterministic criteria, each expert first determines, based on their own experience, the so-called limits aj(e) and bj(e) being the lower and upper limits of the deterministic criterion, respectively [22][34].

Having the limit values, transforming functions, most often linear ones, are prepared, thanks to which it is possible to transform deterministic evaluations into partial values of fuzzy evaluations in the range ⟨0,1⟩ (Figure 3).

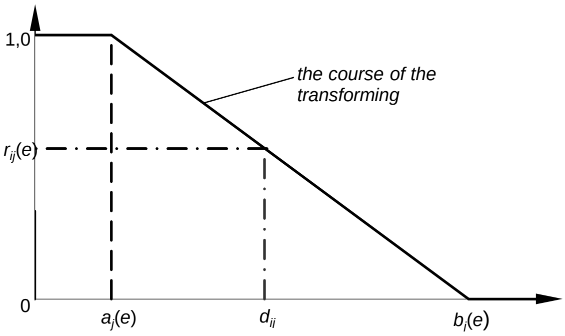

**Fig. 3. T**he scheme of transforming the deterministic evaluation into a partial value of the fuzzy evaluation from the range ⟨0,1⟩: aj(e), e = 1,..., p - the lower limit value of the deterministic criterion determined by the expert, bj(e), e = 1,..., p - upper limit value of the deterministic evaluation determined by the expert, dij - value of the deterministic evaluation of the i-th packaging option in relation to the j-th deterministic criterion, rij(e) - partial value of the fuzzy evaluation determined by transforming the deterministic evaluation.

**Source:** own study

Triangular membership functions are created from the partial values of the fuzzy evaluations rij(e). The quantities rij(min)(d), rij(max)(d), rij(mod)(d) describing the fuzzy evaluation function of the packaging option with respect to the deterministic criterion kj(d) are calculated according to the following formulas (5):

$$\begin{aligned} r_{ij(\min)}^{(d)} &= \min r_{ij}(e) \\ r_{ij(\max)}^{(d)} &= \max r_{ij}(e) \\ r_{ij(\mathrm{mod})}^{(d)} &= \frac{1}{p}\sum_{e=1}^{p} r_{ij}(e) \end{aligned} \tag{5}$$

The method of evaluating the packaging options in the light of fuzzy criteria is almost identical to the evaluation of the importance of individual criteria, in the sense that each expert creates the Saaty matrix of option evaluations by comparing options in pairs against individual fuzzy criteria. The quantities rij(min), rij(max) as well as rij(mod) describing the membership function are calculated according to the following the formulas (6):

$$\begin{aligned} r_{ij(\min)} &= \min \boldsymbol{\lambda}_{ij}(e) \\ r_{ij(\max)} &= \max \boldsymbol{\lambda}_{ij}(e) \\ r_{ij(\mathrm{mod})} &= \frac{1}{p}\sum_{e=1}^{p} \boldsymbol{\lambda}_{ij}(e) \end{aligned} \tag{6}$$

Where in (7):

$$\boldsymbol{\lambda}_{ij}(e) = \frac{\lambda_{ij}(e)}{\max \lambda_{ij}(e)} \tag{7}$$

and λij(e), i = 1,...,n, j = 1,...,m, is the eigenvector corresponding to the largest eigenvalue of the Saaty matrix of option evaluations against fuzzy criteria.

The membership function determining the total fuzzy evaluation of the option ai is in the Baas and Kwakernaak method determined in formula (8) [24]:

$$\mu_{Z_i}(z_i) = sup\ \min\left\{\min\mu_{V_j}(v_j), \min\mu_{R_{ij}}(r_{ij})\right\}, \text{ where: } v_j \in \langle 0,1\rangle,\ r_{ij} \in \langle 0,1\rangle,\ 1 \le j \le m \quad (8)$$

Where (9):

$$z_i = \frac{\sum_{j=1}^{m} v_j r_{ij}}{\sum_{j=1}^{m} v_j} \quad (9)$$

The practical algorithm for determining the membership function of the total fuzzy evaluation µZi(zi) of the i-th option (solution), defined by the formula (8), uses the so-called α–truncations of fuzzy sets. In this way, n fuzzy sets Z1, Z2, ..., Zn are obtained describing the preferences of individual options a1, ..., a2, ..., an. Then, it is necessary to order the fuzzy sets Z1, Z2, ..., Zn according to the assumed ordering relation „≺”, where Zi≺Z means that the option aj is more preferred than the option ai. One of the ordering methods is the method that assigns each of the membership functions µZi its weighted average (10):

$$S_i = \frac{\int_0^1 z\mu_{Z_i}(z)dz}{\int_0^1 \mu_{Z_i}(z)dz} \quad (10)$$

The option as is preferable to at if St < Ss, i.e. Zt≺Zs ⇔ St < Ss. The optimal option is therefore the most preferred option, according to the assumed relation „≺”, i.e. the aopt option for which (11):

$$\max S_i = S^{\text{opt}} \quad (11)$$

The algorithm for determining the membership function of the total fuzzy evaluation µZi (zi), determined by the relation (8) and the weighted average value assigned to it, is presented in the paper [31].

## 3. Parametric Optimization of the Food Packaging Process

Optimization of the conditions for the implementation of the food packaging operation on a roller machine, called internal parametric optimization, can be carried out with the determination of the function of the test object by performing active experiments. Before proceeding with experimental research, it is necessary to determine the following in advance:

characteristics of the test object, in particular the number of input quantities i, their variability ranges xk(min), xk(max), output values zj,..., zw and acceptable functions of the test object (12):

$$z_j = (x_1, x_2, \dots, x_k, \dots, x_i), \text{ Where: j = 1, 2, w} \quad (12)$$

the purpose of the research.

Applying equation (12) to the packaging process, the output values can be written as follows (13):

$$\begin{aligned} W_p &= f\left(t_f, t_z, p_k\right) \to \max \\ s_p &= f\left(t_f, t_z, p_k\right) \to \min \end{aligned} \quad (13)$$

where: Wp - capacity of the packaging process, pcs/h; tf - lower foil forming temperature, oC; tz - the welding temperature of the lower foil with the upper foil, oC; pk - the value of the final vacuum during evacuation of pressure inside the package, hPa; sp - the package tightness expressed by the oxygen content in the package, expressed in percentage.

In the case of extremization of the established criteria for optimizing the operating parameters of a rolling machine, it seems appropriate to use a static plan determined by selective multi-factor orthogonal PS/DS–P: α [28].

After conducting the experiment in accordance with the established experimental plan, the statistical analysis of the obtained test results is started. As a result of this analysis, the Pareto optimum is determined. To determine the set of optimal solutions in the Pareto sense, one can use the normalized method of weights. In this method, the selection of weight coefficients is easier, because the individual criteria are in the standardized form and in this way the decision maker becomes independent of the units in which these criteria are expressed. The preference functions P[f(x*)] for this method are given by formula (14) [28; 32]:

$$P[\mathbf{f}(\mathbf{x}^{*})] = \sum_{j=1}^{J} w_j \frac{f_j(\mathbf{x})}{f_j(\mathbf{x}^{\mathrm{opt}})} \tag{14}$$

where: fj(x) - j-th objective function, fj(xopt) - optimal value of the j-th objective function, wj - weight representing the relative importance of the j-th objective.

In order to find the optimal solution from the set of Pareto optimal solutions, a method using a distance function of form (15) [33]:

$$f_{d(i)} = \sqrt{\sum_{j=1}^{m}\left[d_{i(j)} - d_{id(j)}\right]^{2}} \to \min \tag{15}$$

where: di(j) - the normalized value of the j criterion for individual options, did(j) – the normalized value of the j criterion for the ideal point.

The best option from the set of Pareto-optimal options is the one for which the distance function fd(i) reaches the minimum value.

The optimization criteria used to evaluate variants should be independent and their number, especially in the case of determining the Pareto optimal set, should not exceed three.

To assess the quality of packaged products, it is necessary to use subjective criteria, preferably in a fuzzy form, the definition of which requires expert knowledge.

The multi-criteria optimization model does not take into account probabilistic-statistical criteria, such as the consumption of individual elements of the packaging device.

Another limitation of the model is that in assessing the importance of criteria, a collective importance matrix is taken into account, the terms of which are the arithmetic mean of the corresponding terms of individual partial matrices, determined by experts. This creates a risk that in the case of extremely opposing assessments given by two experts, their values may be almost equal, which may falsely suggest the same degree of importance of the criteria.

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## 4. The example of multi-criteria optimization of the structure of food packaging and operating parameters of the packaging machine

The assessment of food packaging options, consisting in the selection of the optimal combination of multilayer film and gas mixture was carried out in the form of a practical experiment in one of the meat processing plants. For the purposes of the research, the MULTIVAC (Germany) R7400 rolling machine was used the operation diagram of which is shown in Figure 4.

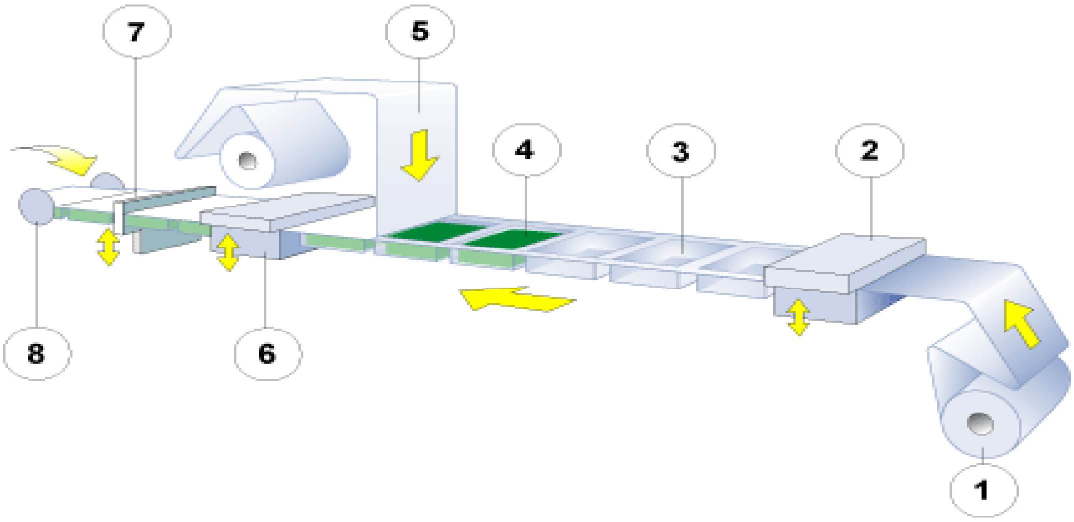


**Fig. 4.** Construction and principle of operation of the rolling machine: 1 - lower foil, 2 - lower foil forming station, 3 - product application area, 4 - product extrusions, 5 - upper foil unwinding station, 6 - welding tool, 7,8 – package cutting unit
**Source:** own study

The product that was packaged in accordance with the accepted set of acceptable options described in subchapter 4.1 was one of the cold cuts. The amount of the product per unit package was 1000 g. Each product came from the same batch of raw material, i.e. it was characterized by the same charge in terms of technology and the same parameters of heat treatment of the raw material related to the manufacturing process. The assumed shelf life of the packed product was 21 days from the date of packaging (manufacturing). In the period between packaging and evaluation, individual packages containing the product were stored in accordance with the manufacturer's recommendations (mainly at the temperature recommended by the manufacturer). After 20 days from the date of packaging, the packaging was opened for evaluation. The settings of the operating parameters of the packaging device were identical for all considered packaging options and resulted from the guidelines specified by the packaging film manufacturers.

### *4.1 A Set of Acceptable Solutions for the Structure of Food Packaging*

The set of acceptable packaging options for food products (cold meats) is presented in Table 1. Determining the set of acceptable packaging options required the following factors to be taken into account:

- The type of packaged product,
- The packaging system,
- The type of packaging device,
- The type of packaging material and medium inside the packaging,
- The expected expiry date,
- The cost of packaging the product.

The list of permissible variants presented in Table 1 is the result (compromise) of the possibilities accompanying the implementation of the experiment in the conditions that existed in an exemplary enterprise operating as a food producer.

**Table 1**
List of acceptable variants of food product packaging

| Option | Type of medium inside the package | Type of film | |
|---|---|---|---|
| | | Upper | Lower |
| *a*₁ | Gas mixture<br>Biogon C10<br>(composition: 10% $CO_2$ + 90% $N_2$) | Amilen 70<br>(composition:<br>polyamide PA 20 µm<br>+ polyethylene PE 50 µm) | Amilen 180<br>(composition:<br>polyamide PA 80 µm<br>+ polyethylene PE 100 µm) |
| *a*₂ | Biogon C10 | Peflex ANP 80<br>(composition:<br>polyamide PA 20 µm)<br>+ polyethylene PE 60 µm) | Peflex ANP 200<br>(composition:<br>polyamide PA 100 µm)<br>+ polyethylene PE 100 µm) |
| *a*₃ | Biogon C10 | Amilen 70 | Peflex ANP 200 |
| *a*₄ | Biogon C10 | Peflex ANP 80 | Amilen 180 |
| *a*₅ | Gas mixture<br>Biogon C20<br>(composition:<br>20% $CO_2$ + 80% $N_2$) | Amilen 70 | Amilen 180 |
| *a*₆ | Biogon C20 | Peflex ANP 80 | Peflex ANP 200 |
| *a*₇ | Biogon C20 | Amilen 70 | Peflex ANP 200 |
| *a*₈ | Biogon C20 | Peflex ANP 80 | Amilen 180 |
| *a*₉ | Gas mixture<br>Biogon C30<br>(composition:<br>30% $CO_2$ + 70% $N_2$) | Amilen 70 | Amilen 180 |
| *a*₁₀ | Biogon C30 | Peflex ANP 80 | Peflex ANP 200 |
| *a*₁₁ | Biogon C30 | Amilen 70 | Peflex ANP 200 |
| *a*₁₂ | Biogon C30 | Peflex ANP 80 | Amilen 180 |

### *4.2 A Set of Criteria for Evaluating the Structure of Food Packaging and Criteria for Evaluating the Operating Parameters of the Packaging Machine*

Due to its complexity and specificity, the food packaging process is assessed based on four groups of criteria, including:

- The sterility and hygiene of the process,
- The quality of the packaged product,
- The quality and aesthetics of the packaging process,
- The cost of the packaging process.

The first of the mentioned groups of criteria (criteria related to sterility and process hygiene) is determined through the prism of regulations and requirements formulated by institutions appropriate for this purpose, including the European Union.

The criteria related to the quality of a packaged product are often based on sensory analysis, the meaning of which comes down to examining the organoleptic characteristics of the product, such as:

- appearance,
- smell,
- deliciousness,
- structure image,

Through the sense organs: taste, smell, touch and sight.

The criteria related to packaging quality concern k, e.g. issues related to the protection of the packaged product, in particular:

- durability and quality of the weld,
- mechanical durability of the packaging foil (impact resistance, pressure resistance and tearing),
- Barrier and tightness in relation to external factors (temperature, water vapour).

However, criteria related to the aesthetics of packaging include protective and information functions, such as:

- Aesthetics of the foil, e.g. in terms of transparency,
- Legibility of the information provided (barcode, expiry date).

Both the criteria relating to the quality of the packaged product and the criteria relating to the quality of the packaging constitute a group of subjective and vague criteria. A separate group of criteria consists of deterministic criteria, which include the cost of product packaging and the efficiency of the packaging process.

To sum up, the proper selection of criteria for assessing variants significantly affects the course of the decision -making process and, as a consequence, the quality of the optimal solution [34].

A set of criteria (Table 2) was adopted for the evaluation of packaging options (Table 1), including both deterministic and fuzzy criteria.

**Table 2**

A set of criteria and methodology for evaluating packaging options

| Criterion | | | Assessment Methodology |
|---|---|---|---|
| No. | Name | Character | |
| $k_1^{(d)}$ | Packing unit cost | Deterministic | Additional calculation (section 4.3.) |
| $k_2^{(d)}$ | Packaging tightness | Deterministic | Measurement of the oxygen content inside the package |
| $k_3^{(r)}$ | Appearance of the packaged product | Fuzzy | Organoleptic evaluation, sensory analysis according to PN-EN ISO 5492:2009 |
| $k_4^{(r)}$ | The smell of the packaged product | Fuzzy | Organoleptic evaluation, sensory analysis according to PN-EN ISO 5492:2009 |
| $k_5^{(r)}$ | The taste of the packaged product | Fuzzy | Organoleptic evaluation, sensory analysis according to PN-EN ISO 5492:2009 |

Source: own study

Parametric optimization carried out in the form of an active experiment in industrial conditions consisted in determining the optimal operating parameters of the rolling machine in relation to:

- the temperature of forming the bottom foil $t_f$,
- the welding temperature of the lower foil with the upper foil $t_z$,
- the value of the final vacuum during the evacuation of the pressure inside the packaging $p_k$
- because of:
- The efficiency of the packaging process $W_p$,
- The package tightness expressed as oxygen content in the package $s_p$.

### *4.3 The selection of the optimal structure of food packaging due to the unit cost and usable quality of food*

To determine the unit cost of packaging in relation to individual options, the add-on calculation algorithm was used according to cost centres. In addition, it was assumed that the calculation of the cost of a unit packaging of a product should take into account both cost components directly related to the packaging process, as well as indirect costs. Therefore, the cost of packing $k_{pak}$ was calculated from formula (16):

$$k_{pak} = k_m + k_p + k_{rec} \tag{16}$$

where: $k_m$ – the material cost of packaging (PLN/pc.); $k_p$ – the cost of the packing process (PLN/pc.); $k_{rec}$ - the cost of packaging recycling related to the so-called product fee (PLN/pc.).

The material cost of the packaging $k_m$ in equation (16) was obtained with the use of equation (17):

$$k_m = k_{mb} + k_{mp} \tag{17}$$

where: $k_{mb} = k_f + k_g$ - the direct material cost of packaging (PLN/pc.); $k_f$ – the unit cost of packaging foil, i.e. upper and lower (PLN/pc.); $k_g$ - the unit cost of the gas mixture (PLN/pc.); $k_{mp} = N_{kmp}k_{mb}$ – the indirect material cost of packaging (PLN/pc.); $N_{kmp}$ – the overhead cost of the indirect material cost (%).

Calculations of the direct material cost of the $k_{mb}$ package for each of the considered options were carried out with the following assumptions regarding the dimensions of the unit package and assuming the weld width equals 5 mm [4]:

- length: 215 mm,
- width: 195 mm,
- height: 120 mm,
- Internal volume: 4.7880 l.

In addition, lower consumption (in terms of width) of the foil constituting the upper part of the packaging (Amilen 70 and Peflex ANP80) was taken into account. Based on the above data, the cost of packaging film per unit of the packaged product was calculated (Table 3).

**Table 3**

Calculations of the cost of packaging film per unit of the packaged product

| No. | Size | Amilen 70 | Peflex ANP80 | Amilen 180 | Peflex ANP200 |
|---|---|---|---|---|---|
| 1 | Foil cost - PLN/m$^2$ | 1,75 | 1,42 | 3,55 | 3,85 |
| | Total film consumption per unit of the packaged product, m$^2$/pc. | | | | |
| 2 | Film dimensions per unit of the packaged product (length × width), m | 0,1900 × 0,2025 | | 0,1900 × 0,2110 | |
| 3 | Total film consumption per unit of the packaged product, m$^2$/pc. | 0,0385 | | 0,0401 | |
| 4 | Cost of foil per unit of the packaged product $k_f$, PLN/pc. | 0,0674 | 0,0547 | 0,1424 | 0,1544 |

Source: own study

Knowing the volume of the unit package, the consumption and cost of the gas mixture were calculated in relation to the unit of the packaged product (Table 4).

**Table 4**

Calculations of the cost of the gas mixture per unit of the packaged product

| No. | Size | Biogon C10 | Biogon C20 | Biogon C30 |
|---|---|---|---|---|
| 1 | The cost of the gas mixture, PLN/m$^3$ | 18,40 | 19,50 | 23,60 |
| 2 | Consumption of the gas mixture per unit of the packaged product, m$^3$/pc. | 0,0048 | 0,0048 | 0,0048 |
| 3 | The cost of the gas mixture per unit of the packaged product kg, PLN/pc. | 0,0881 | 0,0934 | 0,1130 |

Source: own study

Based on the data contained in Tables 3 and 4, the unit material cost of the $k_{mb}$ packaging was determined for individual options (Table 5).

**Table 5**

Calculations of the direct material cost kmb of individual packaging options

| Packaging option | Cost of foil per unit of the packaged product $k_f$ (PLN/pc.) | Cost of the gas mixture per unit of the packaged product $k_g$ (PLN/pc.) | Direct material cost of packaging $k_{mb}$ (PLN/pc.) |
|---|---|---|---|
| | Upper Low | | |
| $a_1$ | 0,06740,1424 | 0,0881 | 0,2978 |
| $a_2$ | 0,05470,1544 | 0,0881 | 0,2972 |
| $a_3$ | 0,06740,1544 | 0,0881 | 0,3099 |
| $a_4$ | 0,05470,1424 | 0,0881 | 0,2851 |
| $a_5$ | 0,06740,1424 | 0,0934 | 0,3031 |
| $a_6$ | 0,05470,1544 | 0,0934 | 0,3024 |
| $a_7$ | 0,06740,1544 | 0,0934 | 0,3151 |
| $a_8$ | 0,05470,1424 | 0,0934 | 0,2904 |

**Table 5**
Calculations of the direct material cost kmb of individual packaging options

| Packaging option | Cost of foil per unit of the packaged product $k_f$ (PLN/pc.) | | Cost of the gas mixture per unit of the packaged product $k_g$ (PLN/pc.) | Direct material cost of packaging $k_{mb}$ (PLN/pc.) |
|---|---|---|---|---|
| | Upper | Low | | |
| $a_9$ | 0,0674 | 0,1424 | 0,1130 | 0,3227 |
| $a_{10}$ | 0,0547 | 0,1544 | 0,1130 | 0,3221 |
| $a_{11}$ | 0,0674 | 0,1544 | 0,1130 | 0,3348 |
| $a_{12}$ | 0,0547 | 0,1424 | 0,1130 | 0,3100 |

Source: own study

Then, the cost of the $k_p$ packing process was calculated in accordance with the add-on calculation scheme. The calculation results are presented in Table 6.

**Table 6**
Calculations of the unit cost of the packing process kp

| Size | Packaging option | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
| $k_p$, zł/szt. | 0,8455 | 1,0424 | 0,8838 | 0,9030 | 0,8455 | 1,0224 | 0,9222 | 0,9222 | 0,8639 | 1,0023 | 0,9222 | 0,9030 |

Source: own study

Assuming that the company is a payer of the product fee, the so-called packaging recycling cost related to the product fee $k_{rec}$ was calculated (Table 7).

**Table 7**
Calculations of the unit cost related to the product fee krec

| Size | Foil type | | | |
|---|---|---|---|---|
| | Amilen 70 | Peflex ANP 80 | Amilen 180 | Peflex ANP 200 |
| Film grammage (g/m$^2$) | 68,85 | 77,60 | 182,90 | 193,90 |
| Unit foil consumption (kg/pc.) | 0,0027 | 0,0030 | 0,0073 | 0,0078 |
| Product fee rate (PLN/kg) | 2,70* | | | |
| Packaging recycling cost Associated with the product fee $k_{rec}$ (PLN/pc.) | 0,0073 | 0,0081 | 0,0197 | 0,0211 |

* Based on the applicable regulation of the Minister of the Environment on the rates of product fees for individual types of packaging.
Source: own study

Assuming a mark-up for the indirect material cost $N_{kmp}$ = 10%, all components of equation (16) were calculated, which finally allowed to determine the unit cost associated with the packaging process $k_{pak}$ (Table 8).

**Table 8**
Calculations of the unit packing cost kpak

| Size | Packaging option | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
| $k_m$, zł/pc | 0,3276 | 0,3269 | 0,3408 | 0,3136 | 0,3334 | 0,3327 | 0,3466 | 0,3194 | 0,3550 | 0,3543 | 0,3682 | 0,3410 |
| $k_p$, zł/pc | 0,8455 | 1,0424 | 0,8838 | 0,9030 | 0,8455 | 1,0224 | 0,9222 | 0,9222 | 0,8639 | 1,0023 | 0,9222 | 0,9030 |
| $k_{rec}$, zł/pc | 0,0270 | 0,0292 | 0,0284 | 0,0278 | 0,0270 | 0,0292 | 0,0284 | 0,0278 | 0,0270 | 0,0292 | 0,0284 | 0,0278 |
| $k_{pak}$, zł/pc | 1,2001 | 1,3985 | 1,2531 | 1,2445 | 1,2059 | 1,3842 | 1,2973 | 1,2695 | 1,2459 | 1,3858 | 1,3189 | 1,2718 |

Source: own study

The calculations show that the solution with the lowest unit packaging cost is the option $a_1$, while the most expensive solution is the option $a_2$.

The packaging options were evaluated by a group of four experts with knowledge and education in the field of food technology:

- the first expert $e_1$ is a person employed as a production manager,
- the second expert $e_2$ is a person employed as a technologist,
- third expert $e_3$ is a person employed as a sales director,
- The fourth expert $e_4$ is the person responsible for quality assurance (internal auditor).

OptDR software was used for efficient calculations.

The subsequent steps of implementing the algorithm with the support of the OptDR software are presented in Table 9.

**Table 9**

Stages of implementing a structural optimization algorithm using software OptDR

| Step # | Description |
|---|---|
| 1 | Entering basic data |
| 2 | Calculation of the criteria importance matrix |
| 3 | Normalization of the coordinates of the eigenvectors of the criteria importance matrix |
| 4 | Creating a membership function for assessing the importance of criteria |
| 5 | Calculation of the variant evaluation matrix against fuzzy criteria |
| 6 | Normalization of the coordinates of the eigenvectors of the variant evaluation matrix against fuzzy criteria |
| 7 | Creation of functions transforming the evaluation of variants in relation to deterministic criteria |
| 8 | Creating a membership function for evaluating fuzzy variants in relation to the adopted criteria |
| 9 | Calculation and normalization of total variant scores |

Source: own study

In order to determine the importance of the criteria adopted for the assessment, each expert built a matrix of the importance of the criteria using the Saaty method (Figure 5).

In the first stage, the input data of the algorithm should be entered in the form of: the number of acceptable variants, the number of experts and the number of criteria (Table 10). Additionally, in the case of evaluation criteria, the type of criterion (deterministic or fuzzy) should be specified.

**Table 10**

Step 1 - entering input data

| Number of | Max | Accepted |
|---|---|---|
| possible variants | 16 | 12 |
| experts | 4 | 4 |
| criteria | 8 | 5 |
| Criterion no. | Name | Type |
| 1 | unit cost of packaging | deterministic |
| 2 | tightness of the packaging | deterministic |
| 3 | external appearance of the product packaging | fuzzy |
| 4 | the smell of the product packaging | fuzzy |
| 5 | the taste of the product packaging | fuzzy |

Source: own study

In order to determine the importance of the criteria adopted for the assessment, each expert built a matrix of the importance of the criteria using the Saaty method (Table 11).

**Table 11**
Step 2 - expert criteria importance matrices calculated with the OptDR software

|  | k1 | k2 | k3 | k4 | k5 |  | k1 | k2 | k3 | k4 | k5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Expert no. 1 | 1,0000 | 1,7500 | 2,3333 | 1,7500 | 2,3333 | Expert no. 2 | 1,0000 | 0,6667 | 1,3333 | 1,0000 | 0,8000 |
|  | 0,5714 | 1,0000 | 1,3333 | 1,0000 | 1,3333 |  | 1,5000 | 1,0000 | 2,0000 | 1,5000 | 1,2000 |
|  | 0,4286 | 0,7500 | 1,0000 | 0,7500 | 1,0000 |  | 0,7500 | 0,5000 | 1,0000 | 0,7500 | 0,6000 |
|  | 0,5714 | 1,0000 | 1,3333 | 1,0000 | 1,3333 |  | 1,0000 | 0,6667 | 1,3333 | 1,0000 | 0,8000 |
|  | 0,4286 | 0,7500 | 1,0000 | 0,7500 | 1,0000 |  | 1,2500 | 0,8334 | 1,6666 | 1,2500 | 1,0000 |
|  | k1 | k2 | k3 | k4 | k5 |  | k1 | k2 | k3 | k4 | k5 |
| Expert no. 3 | 1,0000 | 0,8000 | 4,0000 | 2,0000 | 0,5714 | Expert no. 4 | 1,0000 | 1,3333 | 1,0000 | 0,6667 | 4,0000 |
|  | 1,2500 | 1,0000 | 5,0000 | 2,5000 | 0,7142 |  | 0,7500 | 1,0000 | 0,7500 | 0,5000 | 3,0000 |
|  | 0,2500 | 0,2000 | 1,0000 | 0,5000 | 0,1428 |  | 1,0000 | 1,3333 | 1,0000 | 0,6667 | 4,0000 |
|  | 0,5000 | 0,4000 | 2,0000 | 1,0000 | 0,2857 |  | 1,5000 | 2,0000 | 1,5000 | 1,0000 | 6,0000 |
|  | 1,7500 | 1,4000 | 7,0000 | 3,5000 | 1,0000 |  | 0,2500 | 0,3333 | 0,2500 | 0,1667 | 1,0000 |

Source: own study

In the next step of the procedure carried out using the OptDR software, the coordinates of the eigenvectors of the criteria importance matrix are normalized (Table 12).

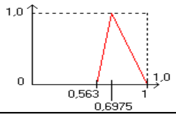
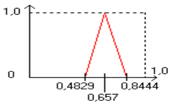
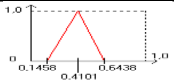
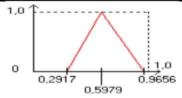
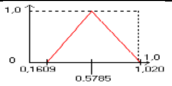
**Table 12**
Step 3 - normalization of the coordinates of the eigenvectors of the criteria importance matrix

| Coordinates of the eigenvector of the criteria validity matrix of expert no. |  | k1 | k2 | k3 | k4 | k5 | k6 | k7 | k8 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | before normalization | 3,5176 | 2,0101 | 1,5076 | 2,0101 | 1,5076 | – | – | – |
|  | after normalization | 1,0000 | 0,5714 | 0,4286 | 0,5714 | 0,4286 | – | – | – |
| 2 | before normalization | 1,9803 | 2,9703 | 1,4853 | 1,9803 | 2,4754 | – | – | – |
|  | after normalization | 0,5630 | 0,8444 | 0,4222 | 0,5630 | 0,7037 | – | – | – |
| 3 | before normalization | 2,0519 | 2,5649 | 0,5130 | 1,0260 | 3,5910 | – | – | – |
|  | after normalization | 0,5833 | 0,7292 | 0,1458 | 0,2917 | 1,0209 | – | – | – |
| 4 | before normalization | 2,2646 | 1,6985 | 2,2646 | 3,3967 | 0,5661 | – | – | – |
|  | after normalization | 0,6438 | 0,4829 | 0,6438 | 0,9656 | 0,1609 | – | – | – |

Source: own study

As a result of normalization of the coordinates of the eigenvectors, triangular membership functions of criteria importance assessments are created (Table 13).

**Table 136**
Step 4 - creating a membership function for assessing the importance of criteria

| Criterion no. 1 |  | Criterion no. 2 |  |
|---|---|---|---|
| v1(min) = 0,5630<br>v1(mod) = 0,6975<br>v1(max) = 1,0000 |  | v2(min) = 0,4829<br>v2(mod) = 0,6570<br>v2(max) = 0,8444 |  |
| Criterion no. 3 |  | Criterion no. 4 |  |
| v3(min) = 0,1458<br>v3(mod) = 0,4101<br>v3(max) = 0,6438 |  | v4(min) = 0,2917<br>v4(mod) = 0,5979<br>v4(max) = 0,9656 |  |
| Criterion no. 5 |  |  |  |
| v5(min) = 0,1609<br>v5(mod) = 0,5785<br>v5(max) = 1,0000 |  |  |  |

Source: own study

The variant evaluation matrices were calculated against the fuzzy criteria in step 5. Table 14 contains an example matrix in which expert $e_1$ assessed the packaging variants against the fuzzy criterion $k_3^{(r)}$.

**Table 14**
Step 5 - Sample evaluation matrix of packaging variants prepared by expert No. 1 against the k3(r) criterion - external appearance of the packaged product

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | 1,0000 | 0,6000 | 1,5000 | 0,7500 | 3,0000 | 1,5000 | 0,6000 | 0,4286 | 0,7500 | 3,0000 | 0,6000 | 1,5000 |
| $a_2$ | 1,6667 | 1,0000 | 2,5000 | 1,2500 | 5,0000 | 2,5000 | 1,0000 | 0,7143 | 1,2500 | 5,0000 | 1,0000 | 2,5000 |
| $a_3$ | 0,6667 | 0,4000 | 1,0000 | 0,5000 | 2,0000 | 1,0000 | 0,4000 | 0,2857 | 0,5000 | 2,0000 | 0,4000 | 1,0000 |
| $a_4$ | 1,3333 | 0,8000 | 2,0000 | 1,0000 | 4,0000 | 2,0000 | 0,8000 | 0,5714 | 1,0000 | 4,0000 | 0,8000 | 2,0000 |
| $a_5$ | 0,3333 | 0,2000 | 0,5000 | 0,2500 | 1,0000 | 0,5000 | 0,2000 | 0,1429 | 0,2500 | 1,0000 | 0,2000 | 0,5000 |
| $a_6$ | 0,6667 | 0,4000 | 1,0000 | 0,5000 | 2,0000 | 1,0000 | 0,4000 | 0,2857 | 0,5000 | 2,0000 | 0,4000 | 1,0000 |
| $a_7$ | 1,6667 | 1,0000 | 2,5000 | 1,2500 | 5,0000 | 2,5000 | 1,0000 | 0,7143 | 1,2500 | 5,0000 | 1,0000 | 2,5000 |
| $a_8$ | 2,3333 | 1,4000 | 3,5000 | 1,7500 | 7,0000 | 3,5000 | 1,4000 | 1,0000 | 1,7500 | 7,0000 | 1,4000 | 3,5000 |
| $a_9$ | 1,3333 | 0,8000 | 2,0000 | 1,0000 | 4,0000 | 2,0000 | 0,8000 | 0,5714 | 1,0000 | 4,0000 | 0,8000 | 2,0000 |
| $a_{10}$ | 0,3333 | 0,2000 | 0,5000 | 0,2500 | 1,0000 | 0,5000 | 0,2000 | 0,1429 | 0,2500 | 1,0000 | 0,2000 | 0,5000 |
| $a_{11}$ | 1,6667 | 1,0000 | 2,5000 | 1,2500 | 5,0000 | 2,5000 | 1,0000 | 0,7143 | 1,2500 | 5,0000 | 1,0000 | 2,5000 |
| $a_{12}$ | 0,6667 | 0,4000 | 1,0000 | 0,5000 | 2,0000 | 1,0000 | 0,4000 | 0,2857 | 0,5000 | 2,0000 | 0,4000 | 1,0000 |

Source: own study

In step 6, the coordinates of the eigenvectors of the variant evaluation matrix are normalized with respect to the fuzzy criteria. Table 15 shows the coordinate values before and after normalization.

**Table 15**
Step 6 – Normalization of the coordinates of the eigenvectors of the variant evaluation matrix against the fuzzy criteria

| Criterion no. | Expert no. | Option | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
| before normalization | | | | | | | | | | | | | |
| $k_3^{(r)}$ | $e_1$ | 2,6908 | 4,4857 | 1,7939 | 3,5878 | 0,8969 | 1,7939 | 4,4847 | 6,2782 | 3,5878 | 0,8969 | 4,4847 | 1,7939 |
| | $e_2$ | 1,6970 | 2,5454 | 4,2425 | 3,3940 | 0,8485 | 2,5454 | 5,0916 | 0,8485 | 4,2425 | 5,9399 | 2,5454 | 3,3940 |
| | $e_3$ | 3,2510 | 3,2510 | 2,4383 | 1,6255 | 4,8762 | 3,2510 | 4,0637 | 2,4383 | 5,6894 | 4,0637 | 1,6255 | 2,4383 |
| | $e_4$ | 1,9531 | 0,9765 | 3,9061 | 2,9295 | 4,8827 | 3,9061 | 2,9295 | 0,9765 | 6,8361 | 3,9061 | 1,9531 | 0,9765 |
| $k_4^{(r)}$ | $e_1$ | 5,1560 | 3,4373 | 4,2966 | 1,7187 | 0,8593 | 2,5780 | 6,0156 | 3,4373 | 2,5780 | 1,7187 | 4,2966 | 0,8593 |
| | $e_2$ | 4,2426 | 2,5456 | 3,3941 | 5,0913 | 1,6970 | 3,3941 | 5,0913 | 4,2426 | 1,6970 | 2,5455 | 3,3941 | 1,6970 |
| | $e_3$ | 4,9103 | 3,2735 | 4,0919 | 3,2735 | 2,4552 | 4,0919 | 5,7290 | 3,2735 | 2,4552 | 0,8184 | 1,6368 | 2,4552 |
| | $e_4$ | 5,7966 | 4,1404 | 3,3123 | 2,4843 | 1,6562 | 0,8281 | 4,9683 | 3,3123 | 4,1404 | 1,6562 | 3,3123 | 2,4843 |
| $k_5^{(r)}$ | $e_1$ | 3,6422 | 4,3709 | 0,7284 | 2,9138 | 3,6422 | 3,1218 | 5,0990 | 4,3709 | 4,3709 | 2,1853 | 1,4569 | 2,9138 |
| | $e_2$ | 2,9711 | 5,1996 | 2,2283 | 1,4855 | 2,9711 | 1,4855 | 4,4564 | 5,1996 | 5,1996 | 1,4855 | 2,9711 | 2,2283 |
| | $e_3$ | 4,9803 | 4,1502 | 3,3202 | 0,8300 | 2,4901 | 0,8300 | 5,8106 | 4,1502 | 4,1502 | 2,4901 | 2,4901 | 1,6601 |
| | $e_4$ | 3,8729 | 4,6477 | 3,8729 | 3,0983 | 3,0983 | 1,5492 | 4,6477 | 4,6477 | 3,8729 | 3,0983 | 1,5492 | 0,7746 |
| after normalization | | | | | | | | | | | | | |
| $k_3^{(r)}$ | $e_1$ | 0,3936 | 0,6560 | 0,2624 | 0,5248 | 0,1312 | 0,2624 | 0,6560 | 0,9184 | 0,5248 | 0,1312 | 0,6560 | 0,2624 |
| | $e_2$ | 0,2482 | 0,3723 | 0,6206 | 0,4965 | 0,1241 | 0,3723 | 0,7448 | 0,1241 | 0,6206 | 0,8689 | 0,3723 | 0,4965 |
| | $e_3$ | 0,4756 | 0,4756 | 0,3567 | 0,2378 | 0,7133 | 0,4756 | 0,5944 | 0,3567 | 0,8323 | 0,5944 | 0,2378 | 0,3567 |
| | $e_4$ | 0,2857 | 0,1428 | 0,5714 | 0,4285 | 0,7143 | 0,5714 | 0,4285 | 0,1428 | 1,0000 | 0,5714 | 0,2857 | 0,1428 |
| $k_4^{(r)}$ | $e_1$ | 0,8571 | 0,5714 | 0,7142 | 0,2857 | 0,1428 | 0,4286 | 1,0000 | 0,5714 | 0,4286 | 0,2857 | 0,7142 | 0,1428 |
| | $e_2$ | 0,7053 | 0,4231 | 0,5642 | 0,8463 | 0,2821 | 0,5642 | 0,8463 | 0,7053 | 0,2821 | 0,4231 | 0,5642 | 0,2821 |
| | $e_3$ | 0,8163 | 0,5442 | 0,6802 | 0,5442 | 0,4081 | 0,6802 | 0,9524 | 0,5442 | 0,4081 | 0,1360 | 0,2721 | 0,4081 |
| | $e_4$ | 0,9636 | 0,6883 | 0,5506 | 0,4130 | 0,2753 | 0,1377 | 0,8259 | 0,5506 | 0,6883 | 0,2753 | 0,5506 | 0,4130 |
| $k_5^{(r)}$ | $e_1$ | 0,6268 | 0,7522 | 0,1254 | 0,5015 | 0,6268 | 0,5373 | 0,8775 | 0,7522 | 0,7522 | 0,3761 | 0,2507 | 0,5015 |
| | $e_2$ | 0,5113 | 0,8948 | 0,3835 | 0,2557 | 0,5113 | 0,2557 | 0,7669 | 0,8948 | 0,8948 | 0,2557 | 0,5113 | 0,3835 |
| | $e_3$ | 0,8571 | 0,7142 | 0,5714 | 0,1428 | 0,4285 | 0,1428 | 1,0000 | 0,7142 | 0,7142 | 0,4285 | 0,4285 | 0,2857 |
| | $e_4$ | 0,6665 | 0,7999 | 0,6665 | 0,5332 | 0,5332 | 0,2666 | 0,7999 | 0,7999 | 0,6665 | 0,5332 | 0,2666 | 0,1333 |

Source: own study

Next, the appointed experts assessed the packaging options in the light of the adopted criteria. In the case of deterministic criteria: unit cost of packaging $k_1^{(d)}$ and packaging tightness $k_2^{(d)}$, it was necessary for each of the experts to adopt the limit values $a_j(e)$ and $b_j(e)$, corresponding to individual criteria. For the deterministic criterion $k_1^{(d)}$, the deterministic evaluations corresponded to the values of the unit packing cost $k_{pak}$ (Table 8). The values of deterministic evaluations in relation to the deterministic criterion $k_2^{(d)}$ were obtained as a result of measuring the oxygen content in the packaging (Table 16) using a gas analyzer.

**Table 16**

Results of measurements of oxygen content in packages, which are deterministic evaluation values for criterion k2(d) (package tightness)

| Size | Option | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
| Oxygen content in the package (%) | 1,2 | 1,4 | 0,7 | 0,8 | 1,1 | 1,3 | 0,6 | 0,8 | 1,1 | 1,3 | 0,8 | 1,0 |

Source: own study

On the basis of the adopted limit values aj(e) and bj(e) and the determined values of assessments of deterministic criteria k1(d) and k2(d), transforming functions were created to model the evaluation of individual packaging variants in the light of these criteria. Figure 5 shows an exemplary transforming function for the deterministic criterion k1(d) created by the expert e1.

Step 7: Creation of functions transforming option evaluations relative to deterministic criteria

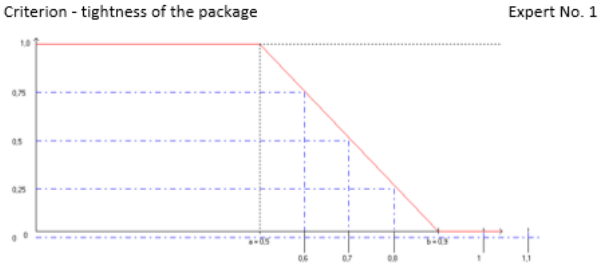


**Fig. 5.** An example of the transforming function for the deterministic criterion k1 (d) created by the expert e1

**Source:** own study

As part of step 8, the experts assessed the packaging options due to the adopted fuzzy criteria which allowed to determine the membership function. Figure 6 presents fuzzy evaluation functions for option a1.

Step 8: Creation of the affinity function of the evaluations of the fuzzy options in relation to the criteria

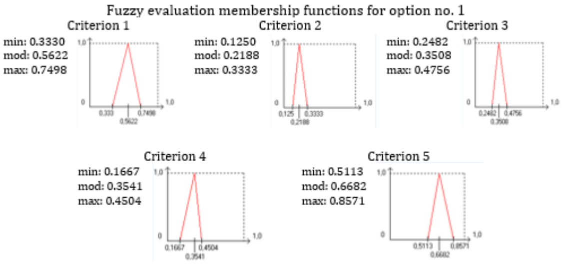


**Fig. 6.** Fuzzy evaluation functions for variant a1 determined with the use of the OptDR software

**Source:** own study

Step 9 consisted in calculation and normalization of total variant scores. The results of the calculations carried out with the use of the OptDR software including the values of the weighted average scores before and after normalization ordering the total membership functions (fuzzy sets), are presented in Table 17.

**Table 17**
Weighted average scores before and after normalization determined using the OptDR software

| The weighted average score | Option | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ |
| before normalization | 0,5309 | 0,3788 | 0,5526 | 0,5051 | 0,4279 | 0,2681 | 0,7113 | 0,6020 | 0,5584 | 0,2867 | 0,4673 | 0,3596 |
| after normalization | 0,7464 | 0,5325 | 0,7769 | 0,7101 | 0,6016 | 0,3769 | **1,0000** | 0,8463 | 0,7850 | 0,4031 | 0,6570 | 0,5056 |

Source: own study

The data in Table 17 shows that the optimal packaging variant is variant a7 with the highest value of the weighted average score equal to 0.7113. This corresponds to the variant of packaging a food product in the upper foil Amilen 70 (composition: polyamide PA 20 µm + polyethylene PE 50 µm) and the lower foil Peflex ANP 200 (composition: polyamide PA 100 µm + polyethylene PE 100 µm) and the medium inside the packaging constituting a protective atmosphere in the form of a gas mixture Biogon C20 (composition: 20% CO2 + 80% N2). The unit cost for this variant amounted to PLN 1.2973 per unit of the packaged product and was approx. 8% higher than the cheapest option.

### *4.4 The selection of the optimal structure of food packaging due to the unit cost and usable quality of food*

The experiment was carried out according to a static design determined by selection multifactorial orthogonal PS/DS-P: α. The symbols used in the tables illustrating the course of calculations were taken from the publication [33]. The sequence of individual tests, the parameters of the packaging process used, the times of the packaging cycle tp, as well as the values of the efficiency of the packaging process Wp and the oxygen content in the package sp are summarized in Table 18.

**Table 18**
The scheme of the tests and the results of process efficiency measurements packaging Wp and tightness of packaging sp

| Plan layout | Repetitions | Tested factors | | | | | | $t_p$, s | Result factors | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_1 = t_f$ °C | $x_2 = t_z$ °C | $x_3 = p_k$ hPa | | $W_p$, pcs./h | | $s_p$, % | |
| | | | | | | | | | $z = W_p$ | $\bar{z} = \overline{W_p}$ | $z = s_p$ | $\bar{z} = \overline{s_p}$ |
| 1 | 1 | –1 | –1 | –1 | 87 | 132 | 7,5 | 46,28 | 155,57 | 156,68 | 0,8 | 0,80 |
| | 2 | | | | | | | 45,46 | 158,38 | | 0,7 | |
| | 3 | | | | | | | 46,13 | 156,08 | | 0,9 | |
| 2 | 4 | +1 | –1 | –1 | 108 | 132 | 7,5 | 52,69 | 136,65 | 134,93 | 0,7 | 0,80 |
| | 5 | | | | | | | 54,12 | 133,04 | | 0,8 | |
| | 6 | | | | | | | 53,29 | 135,11 | | 0,9 | |
| 3 | 7 | –1 | +1 | –1 | 87 | 148 | 7,5 | 47,85 | 150,47 | 153,10 | 0,6 | 0,60 |
| | 8 | | | | | | | 47,06 | 153,00 | | 0,5 | |
| | 9 | | | | | | | 46,20 | 155,84 | | 0,7 | |
| 4 | 10 | +1 | +1 | –1 | 108 | 148 | 7,5 | 54,68 | 131,68 | 132,44 | 0,6 | 0,57 |
| | 11 | | | | | | | 55,13 | 130,60 | | 0,6 | |
| | 12 | | | | | | | 53,32 | 135,03 | | 0,5 | |
| 5 | 13 | –1 | –1 | +1 | 87 | 132 | 11,5 | 45,56 | 158,03 | 157,64 | 0,8 | 0,77 |
| | 14 | | | | | | | 45,09 | 159,68 | | 0,8 | |
| | | | | | | | | | | | 0,9 | |
| | 15 | | | | | | | 46,39 | 155,21 | | 0,7 | |

**Table 18 (Continued)**
The scheme of the tests and the results of process efficiency measurements packaging Wp and tightness of packaging sp

| Plan layout | Repetitions | Tested factors | | | | | | $t_p$, s | Result factors | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_1 = t_f$ °C | $x_2 = t_z$ °C | $x_3 = p_k$ hPa | | $W_p$, pcs./h | | $s_p$, % | |
| | | | | | | | | | $z = W_p$ | $\bar{z} = \overline{W_p}$ | $z = s_p$ | $\bar{z} = \overline{s_p}$ |
| 6 | 16 | +1 | –1 | +1 | 108 | 132 | 11,5 | 51,39 | 140,11 | 139,36 | 0,8 | 0,83 |
| | 17 | | | | | | | 52,46 | 137,25 | | 0,8 | |
| | 18 | | | | | | | 51,16 | 140,73 | | 0,9,, | |
| 7 | 19 | –1 | +1 | +1 | 87 | 148 | 11,5 | 46,54 | 154,71 | 152,97 | 0,6 | 0,57 |
| | 20 | | | | | | | 47,65 | 151,10 | | 0,6 | |
| | 21 | | | | | | | 47,03 | 153,09 | | 0,5 | |
| 8 | 22 | +1 | +1 | +1 | 108 | 148 | 11,5 | 54,28 | 132,65 | 134,26 | 0,6 | 0,53 |
| | 23 | | | | | | | 52,69 | 136,65 | | 0,5 | |
| | 24 | | | | | | | 53,94 | 133,48 | | 0,5 | |
| 9 | 25 | 0 | 0 | 0 | 97,5 | 140 | 9,5 | 51,64 | 139,43 | 139,95 | 0,7 | 0,73 |
| | 26 | | | | | | | 52,57 | 136,96 | | 0,8 | |
| | 27 | | | | | | | 50,19 | 143,45 | | 0,7 | |
| 10 | 28 | $-\alpha$ | 0 | 0 | 85 | 140 | 9,5 | 44,88 | 160,43 | 163,87 | 0,6 | 0,70 |
| | 29 | | | | | | | 43,67 | 164,87 | | 0,8 | |
| | 30 | | | | | | | 43,29 | 166,32 | | 0,7 | |
| 11 | 31 | $+\alpha$ | 0 | 0 | 110 | 140 | 9,5 | 50,68 | 142,07 | 140,99 | 0,8 | 0,70 |
| | 32 | | | | | | | 50,12 | 143,66 | | 0,7 | |
| | 33 | | | | | | | 52,46 | 137,25 | | 0,6 | |
| 12 | 34 | 0 | $-\alpha$ | 0 | 97,5 | 130 | 9,5 | 52,14 | 138,09 | 135,42 | 0,8 | 0,87 |
| | 35 | | | | | | | 53,29 | 135,11 | | 0,9 | |
| | 36 | | | | | | | 54,11 | 133,06 | | 0,9 | |
| 13 | 37 | 0 | $+\alpha$ | 0 | 97,5 | 150 | 9,5 | 54,65 | 131,75 | 131,71 | 0,5 | 0,53 |
| | 38 | | | | | | | 55,12 | 130,62 | | 0,5 | |
| | | | | | | | | | | | 0, | |
| | 39 | | | | | | | 54,23 | 132,77 | | 0,6 | |
| 14 | 40 | 0 | 0 | $-\alpha$ | 97,5 | 140 | 7 | 52,11 | 138,17 | 137,53 | 0,5 | 0,50 |
| | 41 | | | | | | | 51,77 | 139,08 | | 0,4 | |
| | 42 | | | | | | | 53,20 | 135,34 | | 0,6 | |
| 15 | 43 | 0 | 0 | $+\alpha$ | 97,5 | 140 | 12 | 50,20 | 143,43 | 142,80 | 0,6 | 0,53 |
| | 44 | | | | | | | 51,13 | 140,82 | | 0,5 | |
| | 45 | | | | | | | 49,95 | 144,14 | | 0,5 | |

Source: own study

Using the Statistica software and using the data provided in Table 18 the following regression coefficients were calculated: b0, b1, b2, b3, b11, b22, b33, b12, b13 i b23. As a result, the following regression equations were obtained for:

efficiency of the packaging process defined by parameter Wp:

$$W_p = -152{,}58805 - 17{,}30470 t_f + 17{,}06573 t_z + 0{,}04650 p_k + 0{,}08167 t_f^2 - 0{,}06112 t_z^2 + 0{,}07749 p_k^2 + 0{,}00096 t_f t_z + 0{,}03230 t_f p_k - 0{,}02896 t_z p_k \quad (18)$$

Oxygen content in the packaging specified by parameter sp:

$$s_p = 15{,}16801 - 0{,}06147 t_f - 0{,}17567 t_z + 0{,}38980 p_k + 0{,}00044 t_f^2 + 0{,}00066 t_z^2 - 0{,}01877 p_k^2 - 0{,}00020 t_f t_z + 0{,}00040 t_f p_k - 0{,}00052 t_z p_k \quad (19)$$

The statistical analysis of regression equations (18) and (19) is presented in Table 19 and Table 20. The conducted analyzes show that the regression equations (18) and (19) are significant. For relations

(18) and (19) spatial-contour diagrams were prepared (Figure 7 and Figure 8).

**Table 19**
The statistical analysis of the regression equation (18)

| Type of estimation | Number of degrees of freedom | Critical value of the test | Test value | Hypothesis verification |
|---|---|---|---|---|
| Significance of the regression equation (*F* Snedecor test) | $f_1^* = N_b - 1 =$ <br> $= 10 - 1 = 9$ <br> $f_2^* = p_d - N_b =$ <br> $= 45 - 10 = 35$ | $F_{0,05;9;35} = 2{,}161$ | $F_c = 102{,}412$ | $F_c > F_{0,05;9;35}$ the regression equation is significant |
| $R^2 = 0{,}96342$; $R^2$ corrected = 0,95401 | | $\hat{e}_b = 2{,}22678$; $d = 1{,}55\%$ | | $p < 0{,}000000$ |
| Verification of the significance of the function coefficients of the research object | | | | |
| Student's t-test <br> $t_{\alpha;f^{**}}$ for: $\alpha = 0{,}05$; $f^{**} = \sum_{u=1}^{n_d} r_u - N_b = 45-10=35$; $t_{0,05;35} = 2{,}030$ | | | | Severity level <br> $p \le 0{,}05$ |
| Function parameter | Coefficient value | Standard error | *t*-test value(b) | *p*-level |
| free word | –152,58805 | 210,3486 | $t(b_0) = -0{,}725$ | 0,473026 |
| $t_f$ | –17,30470 | 1,3607 | $t(b_1) = -12{,}718$ | $0{,}000001^{-8}$ |
| $t_f^2$ | 0,08167 | 0,0057 | $t(b_{11}) = 14{,}338$ | $0{,}000003^{-10}$ |
| $t_z$ | 17,06573 | 2,6841 | $t(b_2) = 6{,}358$ | $0{,}000003^{-1}$ |
| $t_z^2$ | –0,06112 | 0,0093 | $t(b_{22}) = -6{,}539$ | $0{,}000002^{-1}$ |
| $p_k$ | 0,04650 | 5,3277 | $t(b_3) = 0{,}009$ | 0,993085 |
| $p_k^2$ | 0,07749 | 0,1496 | $t(b_{33}) = 0{,}518$ | 0,607645 |
| $t_f t_z$ | 0,00096 | 0,0054 | $t(b_{12}) = 0{,}178$ | 0,859877 |
| $t_f p_k$ | 0,03230 | 0,0216 | $t(b_{13}) = 1{,}492$ | 0,144566 |
| $t_z p_k$ | –0,02896 | 0,0284 | $t(b_{22}) = -1{,}019$ | 0,315033 |

Note: significant coefficients are marked in bold
Source: own study

**Table 20**
The statistical analysis of regression equation (19)

| Type of estimation | Number of degrees of freedom | Critical value of the test | Test value | Hypothesis verification |
|---|---|---|---|---|
| I-st significance of the regression equation (*F* Snedecor test) | $f_1^* = N_b - 1 =$ <br> $= 10 - 1 = 9$ <br> $f_2^* = p_d - N_b =$ <br> $= 45 - 10 = 35$ | $F_{0,05;9;35} = 2{,}161$ | $F_c = 11{,}447$ | $F_c > F_{0,05;9;35}$ the regression equation is significant |
| $R^2 = 0{,}76642$; $R^2$ corrected = 0,68121 | | $\hat{e}_b = 0{,}07877$; $d = 11{,}78\%$ | | $p < 0{,}000000$ |
| Verification of the significance of the function coefficients of the research object | | | | |
| Student's t-test <br> $t_{\alpha;f^{**}}$ for: $\alpha = 0{,}05$; $f^{**} = \sum_{u=1}^{n_d} r_u - N_b = 45-10=35$; $t_{0,05;35} = 2{,}030$ | | | | Severity level <br> $p \le 0{,}05$ |
| Function parameter | Coefficient value | Standard error | *t*-test value(b) | *p*-level |
| free word | 15,16801 | 7,441117 | $t(b_0) = 2{,}038$ | 0,049121 |
| $t_f$ | –0,06147 | 0,048133 | $t(b_1) = -1{,}277$ | 0,209953 |
| $t_f^2$ | 0,00044 | 0,000202 | $t(b_{11}) = 2{,}175$ | 0,036435 |
| $t_z$ | –0,17567 | 0,094951 | $t(b_2) = -1{,}850$ | 0,072747 |
| $t_z^2$ | 0,00066 | 0,000331 | $t(b_{22}) = 1{,}996$ | 0,053779 |
| $p_k$ | 0,38980 | 0,188470 | $t(b_3) = 2{,}068$ | 0,046071 |
| $p_k^2$ | –0,01877 | 0,005291 | $t(b_{33}) = -3{,}548$ | 0,001127 |
| $t_f t_z$ | –0,00020 | 0,000191 | $t(b_{12}) = -1{,}037$ | 0,307067 |
| $t_f p_k$ | 0,00040 | 0,000766 | $t(b_{13}) = 0{,}518$ | 0,607537 |
| $t_z p_k$ | –0,00052 | 0,001005 | $t(b_{22}) = -0{,}518$ | 0,607537 |

Note: significant coefficients are marked in bold
Source: own study

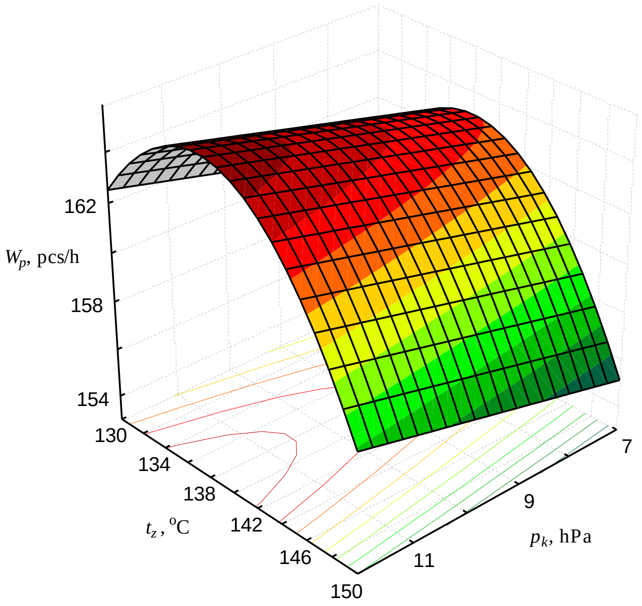


**Fig. 7.** The spatial-contour diagram showing the effect of sealing temperature tz and the value of final vacuum pk on the efficiency of the packaging process defined by parameter Wp, for the optimal value of forming temperature tf = 85 oC
**Source:** own study

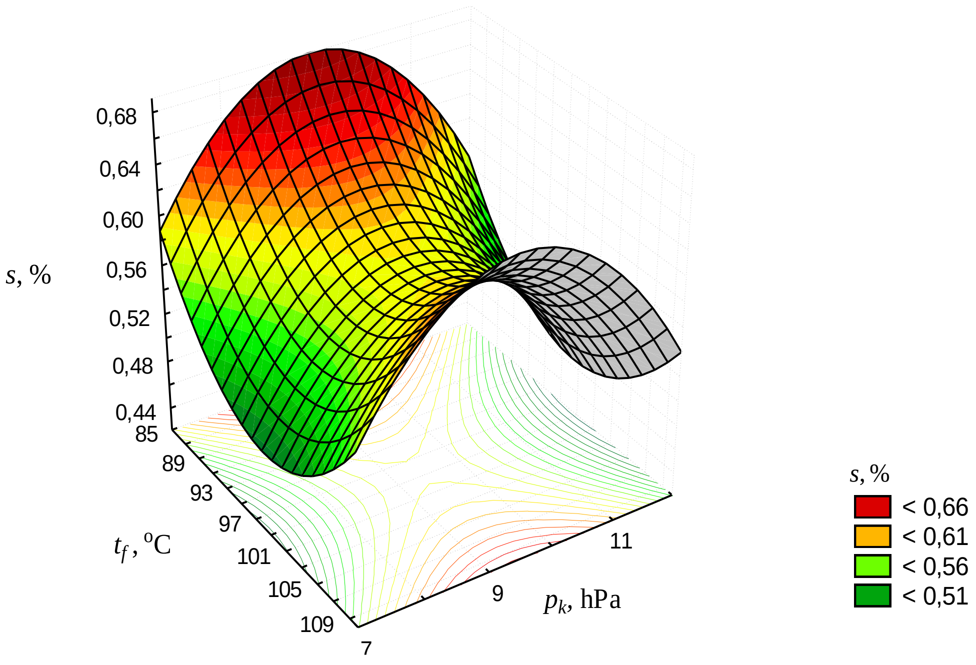


**Fig. 8.** The spatial-contour diagram showing the influence of forming temperature tf and the value of final vacuum pk on the tightness of the package determined by parameter sp, for the optimal value of the sealing temperature tz = 137,43oC
**Source:** own study

For the regression function of complete models defined by equations (18) and (19) extremes were determined using Excel (Solver add-on). The extreme (in this example the maximum) for the efficiency of the packaging process defined by the Wp parameter was observed for the following combination of the tested factors: tf = 85,00 oC, tz = 137,43 oC and pk = 12,00 hPa and equalled: Wp(tf, tz, pk)(max) = 165 pcs/h. The minimum for the oxygen content in the package determined by the parameter sp was observed for the following combination of the tested factors: tf = 98,15 oC, tz = 137,43 oC and pk = 12,00 hPa and equalled: sp(tf, tz, pk)(min) = 0,48%.

The further statistical analysis of the regression equations with non-significant components eliminated was not carried out because complete regression equations are assumed for optimization with respect to 2 objective functions.

A two-stage procedure was used to determine the optimal parameters of the packaging process in terms of two objective functions, i.e. Wp = ϕ(tf, tz, pk) and sp = ϕ(tf, tz, pk). In the first stage, a set of associations of parameters of the packaging process, optimal in the Pareto sense was determined using the standardized method of weights [32] changing the weight values every 0.05. For the single-criterion optimization problem created in this way the form of a substitute criterion was adopted (20):

$$P[\mathbf{f}(\mathbf{x})] = -w_1 \frac{W_p}{W_p^{\mathrm{opt}}} + w_2 \frac{s_p}{s_p^{\mathrm{opt}}} \rightarrow min, \text{ where: w1 + w2 = 1.} \quad (20)$$

Generating a set of Pareto-optimal solutions was carried out with the use of the numerical method implementing the Excel spreadsheet with the Solver add-on. As a result of the calculations, a Pareto set was obtained containing 21 non-dominated solutions due to 2 objective functions (criteria): Wp = ϕ (tf, tz, pk) and sp = ϕ(tf, tz, pk) (Figure 9 and Table 21).

**Table 21**
A set of Pareto-optimal solutions for two objective functions: Wp = ϕ (tf, tz, pk) i sp = ϕ(tf, tz, pk) generated by the weight method

| No. | Input setting values | | | Result factors | |
|---|---|---|---|---|---|
| | $t_f$, °C | $t_z$, °C | $p_k$, hPa | $W_p$, pcs/h | $s_p$, % |
| 1 | 85,00 | 138,62 | 7,00 | 162,87 | 0,57 |
| 2 | 85,00 | 140,35 | 7,00 | 162,68 | 0,55 |
| 3 | 85,00 | 141,57 | 7,00 | 162,34 | 0,53 |
| 4 | 85,00 | 142,46 | 7,00 | 161,96 | 0,52 |
| 5 | 85,00 | 143,15 | 7,00 | 161,61 | 0,52 |
| 6 | 85,00 | 143,70 | 7,00 | 161,29 | 0,52 |
| 7 | 85,00 | 144,15 | 7,00 | 161,00 | 0,51 |
| 8 | 85,00 | 144,51 | 7,00 | 160,74 | 0,51 |
| 9 | 89,31 | 145,08 | 7,00 | 148,67 | 0,46 |
| 10 | 92,96 | 145,58 | 7,00 | 140,77 | 0,42 |
| 11 | 94,80 | 145,94 | 7,00 | 137,51 | 0,41 |
| 12 | 95,91 | 146,23 | 7,00 | 135,74 | 0,41 |
| 13 | 96,65 | 146,47 | 7,00 | 134,62 | 0,41 |
| 14 | 97,18 | 146,68 | 7,00 | 133,83 | 0,41 |
| 15 | 97,58 | 146,86 | 7,00 | 133,24 | 0,40 |
| 16 | 97,89 | 147,02 | 7,00 | 132,78 | 0,40 |
| 17 | 98,14 | 147,16 | 7,00 | 132,40 | 0,40 |
| 18 | 98,34 | 147,29 | 7,00 | 132,09 | 0,40 |
| 19 | 98,51 | 147,41 | 7,00 | 131,82 | 0,40 |
| 20 | 98,66 | 147,51 | 7,00 | 131,59 | 0,40 |
| 21 | 98,78 | 147,61 | 7,00 | 131,38 | 0,40 |

Source: own study

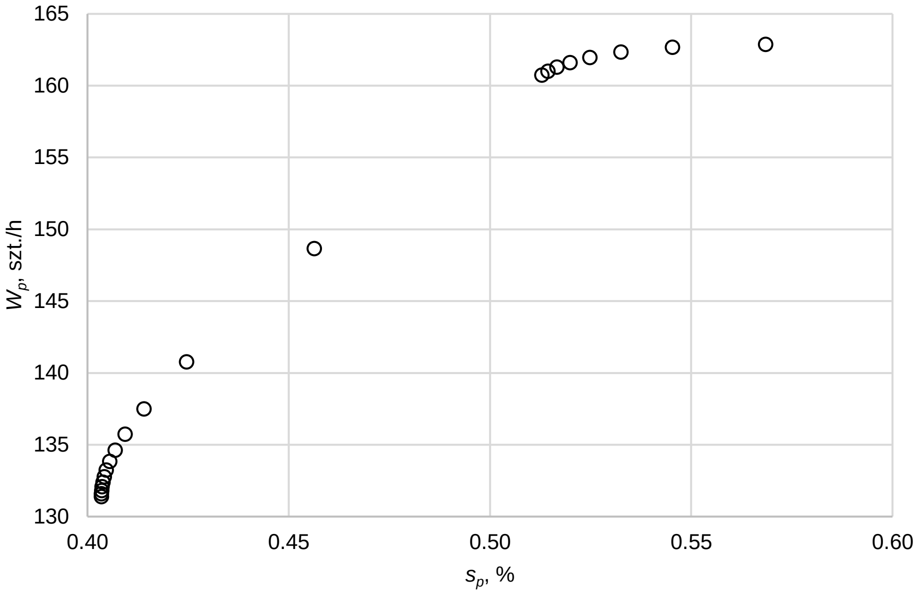


**Fig. 9.** A set of Pareto-optimal solutions for two objective functions: Wp = ϕ (tf, tz, pk) i sp = ϕ (tf, tz, pk)
**Source:** own study

In the second stage, a distance function was used to select the best solution from the Pareto-optimal set. It is assumed that the best solution is the solution from among Pareto-optimal points (non-dominated) closest to the ideal point in the sense of the Euclidean metric. In the case of 2 objective functions Wp = ϕ (tf, tz, pk) and sp = ϕ(tf, tz, pk) the form of the distance function is as follows in (21):

$$d_i[\mathbf{f}(\mathbf{x})] = \sqrt{\left(W_{p(i)}^{(\mathrm{un})} - z_{W_p}^{(\mathrm{id})}\right)^2 + \left(s_{p(i)}^{(\mathrm{un})} - z_{s_p}^{(\mathrm{id})}\right)^2} \tag{21}$$

The values of the set of optimal solutions in the sense of Pareto were normalized by bringing them to the space [0;1] by means of formulas (22) and (23):

$$W_{p(i)}^{(\mathrm{un})} = \frac{W_{p(i)} - \mathrm{min}W_p}{\mathrm{max}W_p - \mathrm{min}W_p} \tag{22}$$

$$s_{p(i)}^{(\mathrm{un})} = \frac{s_{p(i)} - \mathrm{min}s_p}{\mathrm{max}s_p - \mathrm{min}s_p} \tag{23}$$

Then, considering the objective functions Wp = ϕ (tf, tz, pk) and sp = ϕ(tf, tz, pk), maximizing and minimizing them respectively, the coordinates of the ideal point were determined (24):

$$z^{(\mathrm{id})} = \left[\mathrm{max}W_p; \mathrm{min}s_p\right] = [1; 0] \tag{24}$$

The normalized values of the objective function and the values of the distance function di [f(x)] for the Pareto-optimal set are shown in Table 22. The selection of the best solution from the Pareto set using the distance function method is shown in Figure 10.

**Table 22**

Normalized values of distance functions di [f(x)] for the Pareto-optimal set

| No. | $W_{p(i)}^{(un)}$ | $s_{p(i)}^{(un)}$ | $d_i$[f(x)] |
|---|---|---|---|
| 1 | 1,0000 | 1,0000 | 1,0000 |
| 2 | 0,9942 | 0,8597 | 0,8597 |
| 3 | 0,9831 | 0,7819 | 0,7821 |
| 4 | 0,9713 | 0,7352 | 0,7358 |
| 5 | 0,9601 | 0,7055 | 0,7067 |
| 6 | 0,9498 | 0,6858 | 0,6877 |
| 7 | 0,9407 | 0,6724 | 0,6750 |
| 8 | 0,9325 | 0,6629 | 0,6663 |
| 9 | 0,5491 | 0,3206 | 0,5533 |
| 10 | 0,2981 | 0,1283 | 0,7135 |
| 11 | 0,1946 | 0,0640 | 0,8080 |
| 12 | 0,1385 | 0,0355 | 0,8623 |
| 13 | 0,1028 | 0,0208 | 0,8974 |
| 14 | 0,0779 | 0,0124 | 0,9222 |
| 15 | 0,0592 | 0,0074 | 0,9409 |
| 16 | 0,0445 | 0,0042 | 0,9555 |
| 17 | 0,0325 | 0,0023 | 0,9675 |
| 18 | 0,0225 | 0,0011 | 0,9775 |
| 19 | 0,0139 | 0,0004 | 0,9861 |
| 20 | 0,0065 | 0,0001 | 0,9935 |
| 21 | 0,0000 | 0,0000 | 1,0000 |

Source: own study

The best solution is the solution from the Pareto set, for which the value of the distance function di [f(x)] reaches the lowest value, i.e. 0.5533. This corresponds to the following combination of the setting values of the input quantities: the temperature of forming the lower foil tf = 89.31 oC, welding temperature of the lower foil with the upper foil tz = 145.08 oC and the value of the final vacuum during the evacuation of the pressure inside the package pk = 7.00 hPa. For the above combination of setting values, it is possible to achieve optimal values of the resulting factors: efficiency of the packaging process Wpopt = 148.67 ≈ 149 pcs/h and tightness of the packaging expressed by oxygen content in the packaging spopt = 0.46%.

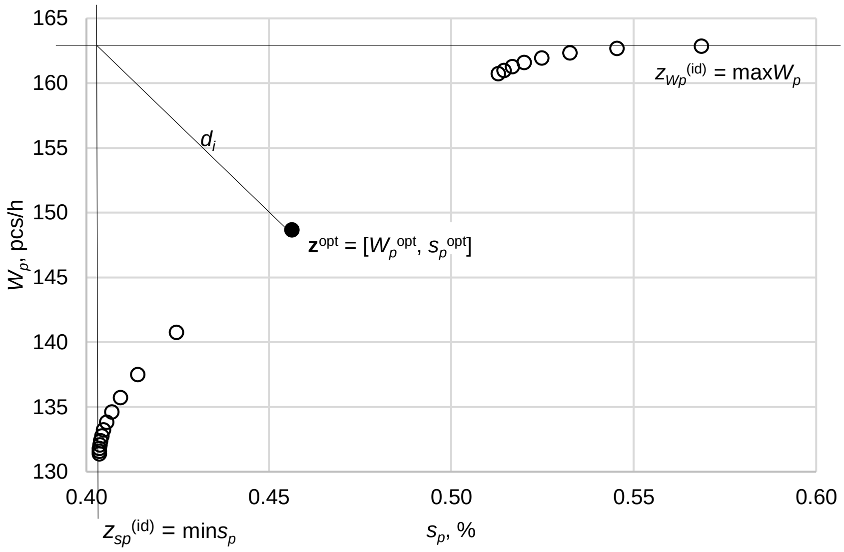


**Fig. 10.** Optimal solution from the Pareto set determined by the distance function method
**Source:** own study

## 5. Conclusions

Multi-criteria optimization of the food packaging process using deterministic criteria and subjective fuzzy criteria gives satisfactory results when its course is two-stage, i.e.:

- in the first stage, the optimal combination of the multi-layer laminate and the medium inside the package in the form of a mixture of protective gas is selected,
- in the second stage, usually in two sub-stages (or steps), the selection of operating parameters of the packaging device is carried out, consisting in determining the set of Pareto optimal solutions due to two objective functions: maximizing process efficiency and minimizing the oxygen content in the package, and then selecting the best solution from this set using the distance function.

In the optimization of the packaging structure, i.e. the type of the film and the medium inside the packaging, in addition to the criterion related to the function providing protection for the packaged product, subjective criteria related to the quality of the packaged product are important, which can be included in the considered problem using fuzzy logic and expert knowledge.

Comparison of the obtained results of structural optimization of food packaging with the results of the experiment using the Yager method gives rise to the conclusion that the optimal packaging configuration includes: Amilen 70 upper foil (composition: polyamide PA 20 µm + polyethylene PE 50 µm) and Peflex lower foil ANP 200 (composition: polyamide PA 100 µm + polyethylene PE 100 µm) and the medium inside the packaging constituting a protective atmosphere in the form of a Biogon C20 gas mixture (composition: 20% CO2 + 80% N2).[4]

When comparing the obtained results regarding parametric optimization with the results of the experiment in which the hierarchical optimization method was used, one should notice their convergence in relation to the following parameters: Wp = 139 pcs/h, sp = 0.5%, tf = 93.36 oC, tz = 146.31 oC and pk = 7 hPa.

To supplement the set of criteria for assessing the packaging process, it would be necessary to take into account the costs resulting from the wear of individual machine components for the adopted settings.

As part of parametric optimization, when generating a set of Pareto-optimal solutions, it would be necessary to use a method based on EC evolutionary calculations - Modified Distance Method (MDM). In turn, in order to select the best solution from the set of Pareto optimal (non-dominated) solutions, it would be justified to use the hierarchical optimization method.

Optimization of the packaging process of food products (cured meat) on thermoforming machine installed in a meat processing plant, performed with respect to two criteria: productivity of packaging process Wp and tightness of the package sp enables selection of optimal parameters (set-ups) of the packaging device, and in effect bring to preserve required protection and quality level of packed product with simultaneous rationalization of costs of the packaging process.

## Author Contributions

Conceptualization, J.P. and R.B. and P.S.; methodology, J.P. and R.B; software, J.P.; validation, J.P. and R.B.; formal analysis, P.S.; investigation, R.B. and P.S.; resources, P.S.; data curation, J.P.; writing—original draft preparation, J.P. and R.B. and P.S.; writing—review and editing, R.B. and P.S.; visualization, J.P.; supervision, J.P.; project administration, J.P. and R.B. and P.S.; funding acquisition, P.S. All authors have read and agreed to the published version of the manuscript.

## Data Availability Statement

In the article, the authors build on the data contained in articles [32] and [27], and all other data used are contained in this article. In case of further requests regarding the data, the authors of the article can be contacted. The authors of this scientific article declare that there is no conflict of interest that could compromise or influence the objectivity, integrity, or results of this study.

## Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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