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SIGNIFICANCE OF TOPSIS APPROACH TO MADM IN COMPUTING EXPONENTIAL DIVERGENCE MEASURES FOR PYTHAGOREAN FUZZY SETS

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Abstract: *Managers nowadays face challenging decisions on daily basis and must weigh a growing number of factors while making such decisions. One of the most common and popular research domains in decision theory is the Multiple Attribute Decision-Making (MADM) problem which allows us to consider several factors into consideration. In this paper, the primary goal is to uncover the important aspect of divergence measures based on exponential function under Pythagorean Fuzzy Sets (PFSs) and study its application to multi attribute decision making. PFSs is a more tensile and powerful approach than intuitionistic fuzzy set (IFS) to depict uncertainty. Numerical example has been illustrated and sensitivity analysis has been carried out to validate our proposed measures. Moreover, a comparative study of the results for the proposed measures demonstrates the efficacy of the proposed distance measures.*

Key words: *Intuitionistic fuzzy set, Pythagorean fuzzy sets, similarity measure, exponential measure, TOPSIS method, multi attribute decision making*

1. Introduction

Evaluations of alternative measures are a challenging and complex task due to several variables which relate to specific decisions in many decision-making problems, such as environmental, social, physical, organizational, and social criteria. Researchers have developed various decision-making methods over the last few years to help policymakers to analyze the strategic planning in the industry and to resolve them. Furthermore, because of the increasing uncertainty and complexity of attributes and the vagueness of human thinking, the study of MADM in an uncertain environment has received much emphasis. For decision-makers, therefore, it is important to

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understand the nature and significance of insecurity to improve their capabilities to take the best decision to decrease risk in their final decisions. Multi-attribute decision-making, a systematic method, can be a useful tool for fuzzy set has its participation and non-participation values totaled to 1. Atanassov (1986, 1989) created the notion of intuitionistic fuzzy set (IFS) to better express uncertainty by easing this constraint. The participation degree and non-participation degree of an IFSs are both real numbers in the range [0, 1], and their sum is less than 1. Another IFSs parameter, the hesitation degree, is derived from the difference between 1 and their summation. IFSs theory has been successfully applied to a variety of real-world challenges such as decision-making. Remarkable outcomes on IFSs have been carried out by many researchers (Peng et al., 2017; Thao et al., 2019).

PFSs is a generalization of IFS that has the prerequisite that the sum of square of perception and non-perception grade ≤ 1. The space of all intuitionistic membership values is also Pythagorean membership values, but not the other way around. Garg (2017) introduced an improved ranking order interval valued PFSs using TOPSIS technique. Indeed, the hypothesis of PFSs has been widely considered, as demonstrated by various researchers (Garg, 2018a, 2018b; Liang & Xu, 2017). In association with the uses of PFSs, Rahman et al. (2017, 2018) proposed a few ways to deal with aggregation operators (AO) and MCDM problems. PFSs have drawn the attention of researchers and are being applied in decision making (Liu et al., 2020; Mahanta & Panda, 2021; Fei & Deng, 2020; Farhadinia, 2021), medical diagnosis (Ejegwa, 2020a, 2020b; Zhou et al., 2020), stock portfolio problem (Khalifa, 2020), belief function (Xiao, 2020). The characteristics and applicability of the measures in pattern recognition, medical diagnosis, multi-criteria decision-making, and clustering analysis were reviewed by Singh and Ganie (2020). Overall, the possibility of PFSs has pulled in incredible considerations of numerous researchers, and the idea has been functional to a few applied regions viz., aggregation operators (Khan et al., 2019), social network analysis (Wang et al., 2020), MCDM (Gao & Wei, 2018; Rahman & Abdullah, 2019), information measures and many more (Yager, 2014; Ejegwa, 2019). Pamučar et al. (2017) investigated the sensitivity of MADM approaches to changes in criteria weight, as well as the methods' consistency in response to changes in measurement scale and created criteria.

A divergence metric for PFSs is a tool that reflects how analogous two or more PFSs are to each other. Indeed, there is a second concept of similarity measurement for PFSs. PFS similarity measures have been investigated from several angles in recent times (Ejegwa, 2018). To address the shortcomings of existing measures, Peng (2019) proposed new Pythagorean distance and similarity measurements (Firozja et al., 2020). Modification of Zhang and Xu’s (2014) distance measure for PFSs and its application to pattern recognition was carried out (Ejegwa, 2020a, 2020b). Some formulae of Pythagorean fuzzy information measures on similarity measures and corresponding transformation relationships were also developed (Peng et al., 2017; Peng & Garg, 2019). Similarity measures for trigonometric function for FSs, IFSs and PFSs were also proposed (Taruna et al., 2021), IFSs and PFSs (Wei & Wei, 2018; Mohd & Abdullah, 2018) were also proposed (Maoying, 2013). Some Complex PFSs distance measures have been established, and their features have been investigated (Ullah et al., 2020). The similarity measures of the IFSs and PFSs are broadly used in various disciplines, comparable to the pattern identification (Peng & Garg, 2019), the clinical finding (Son & Phong, 2016), decision-making (Zhang et al., 2019). However, Lu and Ye (2018) offered similarity measure of IVFSs on log function. Agheli et al. (2022) recently proposed a method for calculating Pythagorean similar measure for two Pythagorean fuzzy value by making use of T-norm and S-norm.

For supplier evaluation and selection, Pamučar et al. (2020) suggested a fuzzy neutrosophic decision-making approach. Many researchers analysed MADM approach using TOPSIS method (Hwang & Yoon, 1981). Many Researchers like Adeel et al. (2019), Akram and Adeel (2019), Akram et al. (2018), Balioti et al. (2018), Biswas and Kumar (2018), Askarifar et al. (2018), Wang and Chen (2017), Gupta et al. (2018), Kumar and Garg (2018) and many more have applied TOPSIS method in various problems of decision making like supplier selection, selection of land, robotics, medical diagnosis, ranking of water quality, human resource selection personnel problem, and many other real life situations flavoured with FSs and generalized FSs.

In this article, we are exploring the resourcefulness of exponential divergence measures of PFSs in the application to pattern recognition and multi attribute decision making. This paper is organized as follows: Section 2 introduces preliminaries of FSs, IFSs and the PFSs. Section 3 comprises of the concept of proposed exponential similarity measures of PFSs. We introduce exponential similarity measures and weighted similarity measures of the PFSs and its numerical computations to validate our measures. Application is also provided in Section 4. Section 5 deliberates discussion about the methodology discussed and sensitivity analysis of the proposed measures. Section 6 compares the new exponential similarity measures with the existing similarity measure by an example. Finally, Section 7 summarizes the document and delivers directions for future experiments.

2. Preliminaries

In this segment, we bring in some basic theories related to FSs, IFSs and PFSs applied in the article.

Definition 2.1. (Zadeh, 1965). Let X be a nonempty set. A fuzzy set P in $E = \{x_1, x_2 \dots, x_n\}$ is characterized by a membership function:

$$P = \{\langle x, \delta_P(x)\rangle | x \in E\} \tag{1}$$

where $\delta_P(x)\colon E \to [0,1]$ is a measure of belongingness of degree of membership of an element $x \in E$ in P.

Definition 2.2. (Atanassov, 1986). An IFS P in X is given by

$$P = \{\langle x, \delta_P(x),\ \zeta_P(x)\rangle | x \in E\} \tag{2}$$

where $\delta_P(x), \zeta_P(x)\colon E \to [0,1]$, $0 \le \delta_P(x) + \zeta_P(x) \le 1, \forall\, x \in E$. The number $\delta_P(x)$ and $\zeta_P(x)$ represents, respectively, the membership degree and non-membership degree of the element x to the set P.

For each IFS P in E, if

$$\eta_P(x) = 1 - \delta_P(x) - \zeta_P(x), \forall\, x \in E. \tag{3}$$

Then $\eta_P(x)$ is called the degree of indeterminacy of x to Ã.

Definition 2.3. (Yager, 2013a, 2013,b). An IFS P in E is given by

$$P = \{\langle x, \delta_P(x),\ \zeta_P(x)\rangle | x \in E\}$$

where $\delta_P(x), \zeta_P(x)\colon E \to [0,1]$, with the condition that

$$0 \le \delta_P^2(x) + \zeta_P^2(x) \le 1, \forall\, x \in E \tag{4}$$

and the degree of indeterminacy for any PFS P and $x \in E$ is given by

$$\eta_P^2(x) = \sqrt{1 - \delta_P^2(x) - \zeta_P^2(x)} \tag{5}$$

3. Exponential Divergence Measures

In this segment, new exponential divergence measures of the PFSs are proposed.

Preposition 1. Let X be nonempty set and P, Q, R ∈ PFS (X). The divergence measure between P and Q is a function $Div: PFS \times PFS \rightarrow [0,1]$ satisfies

(P1) Boundedness: $0 \leq Div(P,Q) \leq 1$

(P2) Separability: $Div(P,Q) = 0 \Leftrightarrow P = Q$.

(P3) Symmetric: $Div(P,Q) = Div(Q,P)$

(P4) Inequality: If R is a PFS in X and $P \subseteq Q \subseteq R$, then $Div(P,Q) \leq Div(P,R)$ and $Div(Q,R) \leq Div(P,R)$.

In several circumstances, the weight of the elements $x_i \in X$ must be considered. For instance, in decision making, the attributes usually have distinct significance, and thus ought to be designated unique weights. As a result, we propose two weighted logarithmic divergence measures between P and Q, as follows:

Let $P, Q \in$ PFS (X) such that $X = \{x_1, x_2 \dots, x_n\}$ then

$$D_{PFSE}(P,Q) = \frac{2}{n}\sum_{i=1}^{n}\left[\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|.\,2^{-\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|.\,2^{-\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|-1}\right] \tag{6}$$

$$D_{WPFSE}(P,Q) = \frac{2}{n}\sum_{i=1}^{n}\omega_i\left[\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|.\,2^{-\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|.\,2^{-\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|-1}\right] \tag{7}$$

$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $x_i (i = 1,2,\dots,n)$, with $\omega_k \in [0,1]$, $k = 1,2,\dots,n$, $\sum_{k=1}^{n}\omega_k = 1$. If $\omega = \left(\frac{1}{n}, \frac{1}{n}, \dots \frac{1}{n}\right)^T$, then the weighted exponential divergence measure reduces to proposed measure. If we take $\omega_k = 1$, $k = 1,2,\dots,n$, then then $D_{WPFSE}(P,Q) = D_{PFSE}(P,Q)$.

Theorem 3.1. The Pythagorean fuzzy divergence measures defined in equation (6) - (7) are valid measures of Pythagorean fuzzy divergence.

Proof. All the necessary four conditions to be a divergence measure are satisfied by the new divergence measures as follows:

(P1) Boundedness: $0 \leq D_{PFSE}(P,Q) \leq 1$

Proof. Since the values $0 \leq \delta_P(x_i) \leq \delta_Q(x_i) \leq 1$ and $0 \leq \zeta_P(x_i) \leq \zeta_Q(x_i) \leq 1$, therefore, $0 \leq \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| \leq 1$ and $0 \leq \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right| \leq 1$. Since minimum values of all the expression is 0, then the measure $D_{PFSL}(P,Q)$ will have value as $D_{PFSE}(P,Q) = \frac{2}{1}(0\,.\,2^{-1} + 0\,.\,2^{-1}) = 0$. Also, if the maximum value of the above expressions is 1, then $D_{PFSL}(P,Q) = \frac{2}{1}(1\,.\,2^{-2} + 1\,.\,2^{-2}) = 1$.

Thus, $0 \leq D_{PFSE}(P,Q) \leq 1$.

Measure $D_{WPFSE}(P,Q)$ can be proved similarly.

(P2) Separability: $D_{PFSE}(P,Q) = 0 \Leftrightarrow P = Q$.

Proof. For two PFSs P and Q in $X = \{x_1, x_2 \dots, x_n\}$, if $P = Q$, then $\delta_P^2(x_i) = \delta_Q^2(x_i)$ and $\zeta_P^2(x_i) = \zeta_Q^2(x_i)$. Thus, $\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| = 0$ and $\left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right| = 0$.

Therefore, $D_{PFSE}(P,Q) = 0$.

If $D_{PFSE}(P,Q) = 0$, this implies

$$\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|.\,2^{-\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|.\,2^{-\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|-1} = 0$$

$\Rightarrow \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| = 0$ and $\left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right| = 0$.

Therefore $\delta_P^2(x_i) = \delta_Q^2(x_i)$ and $\zeta_P^2(x_i) = \zeta_Q^2(x_i)$. Hence P = Q.
Measure $D_{WPFSE}(P, Q)$ can be proved similarly.

(P3) Symmetric: $D_{PFSE}(P, Q) = D_{PFSE}(Q, P)$
Proofs are self-explanatory and straight forward.

(P4) Inequality: If R is a PFS in X and $P \subseteq Q \subseteq R$, then $D_{PFSE}(P, Q) \leq D_{PFSE}(P, R)$ and $D_{PFSE}(Q, R) \leq D_{PFSE}(P, R)$.
Proof. If $P \subseteq Q \subseteq R$, then for $x_i \in X$, we have $0 \leq \delta_P(x_i) \leq \delta_Q(x_i) \leq \delta_R(x_i) \leq 1$ and $1 \geq \zeta_P(x_i) \geq \zeta_Q(x_i) \geq \zeta_R(x_i) \geq 0$.
This implies that $0 \leq \delta_P^2(x_i) \leq \delta_Q^2(x_i) \leq \delta_R^2(x_i) \leq 1$ and $1 \geq \zeta_P^2(x_i) \geq \zeta_Q^2(x_i) \geq \zeta_R^2(x_i) \geq 0$. Thus, we have,
$\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| \leq |\delta_P^2(x_i) - \delta_R^2(x_i)|$; $\left|\delta_Q^2(x_i) - \delta_R^2(x_i)\right| \leq |\delta_P^2(x_i) - \delta_R^2(x_i)|$ and
$\left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right| \leq |\zeta_P^2(x_i) - \zeta_R^2(x_i)|$; $\left|\zeta_Q^2(x_i) - \zeta_R^2(x_i)\right| \leq |\zeta_P^2(x_i) - \zeta_R^2(x_i)|$
From the above we can write, $2^{-|\delta_P^2(x_i)-\delta_Q^2(x_i)|-1} \leq 2^{-|\delta_P^2(x_i)-\delta_R^2(x_i)|-1}$
$\Rightarrow \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|. 2^{-|\delta_P^2(x_i)-\delta_Q^2(x_i)|-1} \leq |\delta_P^2(x_i) - \delta_R^2(x_i)|. 2^{-|\delta_P^2(x_i)-\delta_R^2(x_i)|-1}$
$\Rightarrow \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|. 2^{-|\delta_P^2(x_i)-\delta_Q^2(x_i)|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|. 2^{-|\zeta_P^2(x_i)-\zeta_Q^2(x_i)|-1}$
$\leq |\delta_P^2(x_i) - \delta_R^2(x_i)|. 2^{-|\delta_P^2(x_i)-\delta_R^2(x_i)|-1} + |\zeta_P^2(x_i) - \zeta_R^2(x_i)|. 2^{-|\zeta_P^2(x_i)-\zeta_R^2(x_i)|-1}$
$$\Rightarrow \frac{2}{n}\left[\sum_{i=1}^{n}\left\{\left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|. 2^{-|\delta_P^2(x_i)-\delta_Q^2(x_i)|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|. 2^{-|\zeta_P^2(x_i)-\zeta_Q^2(x_i)|-1}\right\}\right]$$
$$\leq \frac{2}{n}\left[\sum_{i=1}^{n}\left\{|\delta_P^2(x_i) - \delta_R^2(x_i)|. 2^{-|\delta_P^2(x_i)-\delta_R^2(x_i)|-1} + |\zeta_P^2(x_i) - \zeta_R^2(x_i)|. 2^{-|\zeta_P^2(x_i)-\zeta_R^2(x_i)|-1}\right\}\right]$$
$\Rightarrow D_{PFSE}(P, Q) \leq D_{PFSE}(P, R)$. Similarly, $D_{PFSE}(Q, R) \leq D_{PFSE}(P, R)$.
Similar proofs can be made for $D_{WPFSL}(P, Q) \leq D_{WPFSE}(P, R)$ and $D_{WPFSE}(Q, R) \leq D_{WPFSE}(P, R)$.

3.1. Numerical Verification of the Distance Measures

Based on the parameters suggested by Wei and Wei (2018), we verify whether proposed divergence measures satisfy above four properties:
Example 1. *Let $P, Q, R \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$. Suppose*
$P = \{\langle x_1, 0.6, 0.2\rangle, \langle x_2, 0.4, 0.6\rangle, \langle x_3, 0.5, 0.3\rangle\}$,
$Q = \{\langle x_1, 0.8, 0.2\rangle, \langle x_2, 0.7, 0.3\rangle, \langle x_3, 0.6, 0.3\rangle\}$ *and*
$R = \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.8, 0.2\rangle, \langle x_3, 0.7, 0.1\rangle\}$
Calculating the distance using proposed distance measures are as follows:
$$D_{PFSE}(P, Q) = \frac{2}{3}\begin{bmatrix} \left\{|0.6^2 - 0.8^2|. 2^{-|0.6^2-0.8^2|-1} + |0.2^2 - 0.2^2|. 2^{-|0.2^2-0.2^2|-1}\right\} + \\ \left\{|0.4^2 - 0.7^2|. 2^{-|0.4^2-0.7^2|-1} + |0.6^2 - 0.3^2|. 2^{-|0.6^2-0.3^2|-1}\right\} + \\ \left\{|0.5^2 - 0.6^2|. 2^{-|0.5^2-0.6^2|-1} + |0.3^2 - 0.3^2|. 2^{-|0.3^2-0.3^2|-1}\right\} \end{bmatrix}$$
$$= \frac{2}{3}[0.115302742 + 0 + 0.131263519 + 0.111958138 + 0.050962343 + 0]$$
$= 0.272991161.$
$$D_{PFSE}(P, R) = \frac{2}{3}\begin{bmatrix} \left\{|0.6^2 - 0.9^2|. 2^{-|0.6^2-0.9^2|-1} + |0.2^2 - 0.1^2|. 2^{-|0.2^2-0.1^2|-1}\right\} + \\ \left\{|0.4^2 - 0.8^2|. 2^{-|0.4^2-0.8^2|-1} + |0.6^2 - 0.2^2|. 2^{-|0.6^2-0.2^2|-1}\right\} + \\ \left\{|0.5^2 - 0.7^2|. 2^{-|0.5^2-0.7^2|-1} + |0.3^2 - 0.1^2|. 2^{-|0.3^2-0.1^2|-1}\right\} \end{bmatrix}$$

$= \frac{2}{3}[0.16470964 + 0.014691304 + 0.172074629 + 0.12817118 + 0.101609437 + 0.037842305] = 0.41273233.$

$$D_{PFSE}(Q,R) = \frac{2}{3}\begin{bmatrix} \{|0.8^2 - 0.9^2|.\,2^{-|0.8^2-0.9^2|-1} + |0.2^2 - 0.1^2|.\,2^{-|0.2^2-0.1^2|-1}\} + \\ \{|0.7^2 - 0.8^2|.\,2^{-|0.7^2-0.8^2|-1} + |0.3^2 - 0.2^2|.\,2^{-|0.3^2-0.2^2|-1}\} + \\ \{|0.6^2 - 0.7^2|.\,2^{-|0.6^2-0.7^2|-1} + |0.3^2 - 0.1^2|.\,2^{-|0.3^2-0.1^2|-1}\} \end{bmatrix}$$

$= \frac{2}{3}[0.075551627 + 0.014691304 + 0.067593784 + 0.024148408 + 0.05939904 + 0.037842305] = 0.186150981.$

The detailed computation for the proposed measures can be summarized in the table 1:

Table 1. Numerical illustration to validate proposed measures

Proposed Measure 1	*Numerical Values*	*Proposed Measure 2*	*Numerical Values*
$D_{PFSE}(P,Q)$	0.272991	$D_{WPFSE}(P,Q)$	0.093874
$D_{PFSE}(P,R)$	0.412732	$D_{WPFSE}(P,R)$	0.138443
$D_{PFSE}(Q,R)$	0.186151	$D_{WPFSE}(Q,R)$	0.061395

From the above computations, it supports that $D_{PFSE}(P,Q) \leq D_{PFSE}(P,R)$ and $D_{PFSE}(Q,R) \leq D_{PFSE}(P,R)$. Also, $D_{WPFSE}(P,Q) \leq D_{WPFSE}(P,R)$ and $D_{WPFSE}(Q,R) \leq S_{WPFSE}(P,R)$.

4. Applications of Exponential Divergence Measures

One of the most crucial aspects of building model is deciding on criteria. As a result, criteria are critical components that allow options to be compared from a particular perspective. Users are often satisfied with a product when its characteristics fit their tastes and expectations. The most essential product selection criteria for consumers must be determined to design an effective decision model.

PFSs has been frequently used to handle multi attribute decision making (MADM) problems in Pythagorean fuzzy environments due to its excellent skill in representing uncertain information. Many PFSs based MADM algorithms have been proposed. We are presenting a novel Pythagorean fuzzy MADM approach based on the TOPSIS method in this part. Under the Pythagorean fuzzy condition, MADM problem can be depicted by a flowchart as shown in Figure 1.

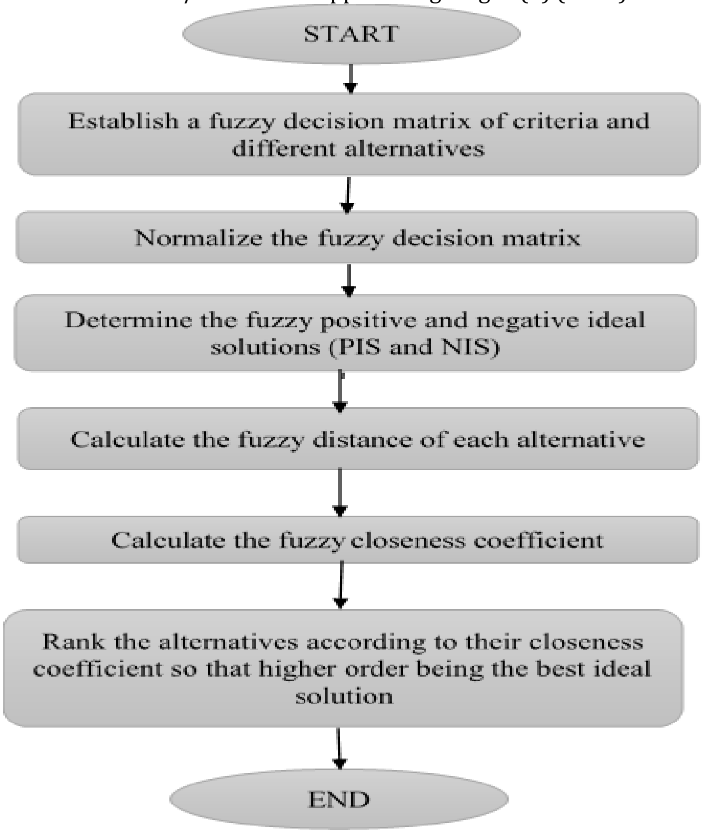

Figure 1. TOPSIS Algorithm Approach

4.1. A Case Study

A multinational company wishes to buy smartphones for their white-collar workers. Because of the large number of smart phones to be acquired, the process is crucial. With members as Procurement Manager ($Ɗ_{\mathbf{1}}$), Human Resource Manager ($Ɗ_{\mathbf{2}}$) and Quality Manager ($Ɗ_{\mathbf{3}}$), a decision committee of three experts is formed by the company with the goal of determining the most appropriate smartphone among five possible options as Model 1 ($\mathcal{A}_1$), Model 2 ($\mathcal{A}_2$), Model 3 ($\mathcal{A}_3$), Model 4 ($\mathcal{A}_4$) and Model 5 ($\mathcal{A}_5$). Experts assist in the decision-making process and employ smartphone choosing criteria. There are five options available as storage capacity in gigabytes($\mathbb{C}_{\mathbf{1}}$), weight in grams($\mathbb{C}_{\mathbf{2}}$), camera specifications in pixels($\mathbb{C}_{\mathbf{3}}$), screen size in inches($\mathbb{C}_{\mathbf{4}}$), battery life in hours($\mathbb{C}_{\mathbf{5}}$). These models were chosen due to their similar costs in the Indian market. Organization assigns weights to these criteria as ω =0.25, 0.35, 0.20, 0.12, and 0.08, respectively.

Step 1: Establish the decision matrix (X)

For each criterion, the options are first examined by the decision makers $Ɗ_{\mathbf{1}}$, $Ɗ_{\mathbf{2}}$ and $Ɗ_{\mathbf{3}}$ using pair-wise comparisons. The assessment of the given alternatives in the form of Pythagorean fuzzy sets by these three decision makers are examined in table 2-4.

Table 2. Data set in the form of a decision matrix (X) of decision maker $\mathcal{D}_1$

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathcal{D}_1$	$\mathcal{A}_1$	<0.8,0.1>	<0.1,0.6>	<0.2,0.8>	<0.6,0.1>	<0.1,0.6>
	$\mathcal{A}_2$	<0.0,0.8>	<0.4,0.4>	<0.6,0.3>	<0.1,0.7>	<0.1,0.8>
	$\mathcal{A}_3$	<0.6,0.1>	<0.4,0.5>	<0.3,0.0>	<0.7,0.2>	<0.3,0.4>
	$\mathcal{A}_4$	<0.7,0.3>	<0.3,0.4>	<0.7,0.2>	<0.8,0.1>	<0.2,0.5>
	$\mathcal{A}_5$	<0.5,0.3>	<0.5,0.4>	<0.7,0.2>	<0.6,0.1>	<0.4,0.7>

Table 3. Data set in the form of a decision matrix (X) of decision maker $\mathcal{D}_2$

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathcal{D}_2$	$\mathcal{A}_1$	<0.4,0.0>	<0.0,0.7>	<0.3,0.3>	<0.1,0.8>	<0.4,0.0>
	$\mathcal{A}_2$	<0.3,0.5>	<0.6,0.2>	<0.6,0.1>	<0.2,0.4>	<0.3,0.5>
	$\mathcal{A}_3$	<0.1,0.7>	<0.9,0.0>	<0.2,0.7>	<0.8,0.0>	<0.1,0.7>
	$\mathcal{A}_4$	<0.4,0.3>	<0.8,0.1>	<0.2,0.6>	<0.2,0.7>	<0.4,0.3>
	$\mathcal{A}_5$	<0.4,0.5>	<0.5,0.3>	<0.8,0.2>	<0.7,0.3>	<0.3,0.6>

Table 4. Data set in the form of a decision matrix (X) of decision maker $\mathcal{D}_3$

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathcal{D}_3$	$\mathcal{A}_1$	<0.2,0.6>	<0.8,0.3>	<0.4,0.5>	<0.1,0.7>	<0.6,0.5>
	$\mathcal{A}_2$	<0.6,0.3>	<0.5,0.3>	<0.6,0.3>	<0.5,0.2>	<0.2,0.6>
	$\mathcal{A}_3$	<0.7,0.2>	<0.6,0.2>	<0.7,0.2>	<0.6,0.3>	<0.3,0.4>
	$\mathcal{A}_4$	<0.3,0.8>	<0.2,0.6>	<0.3,0.6>	<0.6,0.3>	<0.4,0.8>
	$\mathcal{A}_5$	<0.2,0.6>	<0.8,0.3>	<0.4,0.5>	<0.1,0.7>	<0.6,0.5>

Step 2: Calculation of Normalized decision matrix (X)

In the crisp environment, to avoid the complicated normalization formula used in classical TOPSIS, simpler formulas are used to transform the various criteria scales into a comparable scale. $\mathbb{C}_1$, $\mathbb{C}_3$, and $\mathbb{C}_4$ are benefit criteria, while $\mathbb{C}_2$ is cost qualities, according to these experts. However, in case of Pythagorean fuzzy environment, normalized matrix can be constructed by replacing membership and non-membership values in cost attributes, whereas there will not be any change in case of benefit attributes. The results are shown in table 5-8.

Table 5. Normalized values of decision maker $\mathcal{D}_1$ in terms of PFSs (X)

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathcal{D}_1$	$\mathcal{A}_1$	<0.8,0.1>	<0.6,0.1>	<0.2,0.8>	<0.6,0.1>	<0.1,0.6>
	$\mathcal{A}_2$	<0.0,0.8>	<0.4,0.4>	<0.6,0.3>	<0.1,0.7>	<0.1,0.8>
	$\mathcal{A}_3$	<0.6,0.1>	<0.5,0.4>	<0.3,0.0>	<0.7,0.2>	<0.3,0.4>
	$\mathcal{A}_4$	<0.7,0.3>	<0.4,0.3>	<0.7,0.2>	<0.8,0.1>	<0.2,0.5>
	$\mathcal{A}_5$	<0.5,0.3>	<0.4,0.5>	<0.7,0.2>	<0.6,0.1>	<0.4,0.7>

Table 6. Normalized values of decision maker $\mathfrak{D}_2$ in terms of PFSs (X)

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
	$\mathcal{A}_1$	<0.4,0.0>	<0.7,0.0>	<0.3,0.3>	<0.1,0.8>	<0.4,0.0>
	$\mathcal{A}_2$	<0.3,0.5>	<0.2,0.6>	<0.6,0.1>	<0.2,0.4>	<0.3,0.5>
$\mathfrak{D}_2$	$\mathcal{A}_3$	<0.1,0.7>	<0.0,0.9>	<0.2,0.7>	<0.8,0.0>	<0.1,0.7>
	$\mathcal{A}_4$	<0.4,0.3>	<0.1,0.8>	<0.2,0.6>	<0.2,0.7>	<0.4,0.3>
	$\mathcal{A}_5$	<0.4,0.5>	<0.3,0.5>	<0.8,0.2>	<0.7,0.3>	<0.3,0.6>

Table 7. Normalized values of decision maker $\mathfrak{D}_3$ in terms of PFSs (X)

Decision Maker	Alternatives vs Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
	$\mathcal{A}_1$	<0.2,0.6>	<0.3,0.8>	<0.4,0.5>	<0.1,0.7>	<0.6,0.5>
	$\mathcal{A}_2$	<0.6,0.3>	<0.3,0.5>	<0.6,0.3>	<0.5,0.2>	<0.2,0.6>
$\mathfrak{D}_3$	$\mathcal{A}_3$	<0.7,0.2>	<0.2,0.6>	<0.7,0.2>	<0.6,0.3>	<0.3,0.4>
	$\mathcal{A}_4$	<0.3,0.8>	<0.6,0.2>	<0.3,0.6>	<0.6,0.3>	<0.4,0.8>
	$\mathcal{A}_5$	<0.2,0.6>	<0.3,0.8>	<0.4,0.5>	<0.1,0.7>	<0.6,0.5>

Step 3: Identify the Fuzzy Positive Ideal Solution (FPIS) and Negative Ideal Solution (FNIS)

FPIS maximizes the benefit and minimizes the cost, whereas the FNIS maximizes the cost and minimizes the benefit. For each decision maker, we compute FPIS and FNIS for the PFSs using

$$A^{k+} = \{r_1^{k+}, r_2^{k+}, \dots, r_n^{k+}\} = \left\{\left(\max_i(r_{ij}^k) \;/\; j \in I\right), \left(\left(\min_i(r_{ij}^k) \;/\; j \in J\right)\right)\right\}; \tag{8}$$

$$A^{k-} = \{r_1^{k-}, r_2^{k-}, \dots, r_n^{k-}\} = \left\{\left(\min_i(r_{ij}^k) \;/\; j \in I\right), \left(\left(\max_i(r_{ij}^k) \;/\; j \in J\right)\right)\right\} \tag{9}$$

where I refer to the benefit criteria and J, the cost criteria. The subsequent values are presented in table 8.

Table 8. Fuzzy Positive and Negative Ideals for each decision makers

Decision Maker	FPIS and FNIS	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
	A^+	<0.8,0.1>	<0.6,0.1>	<0.7,0.0>	<0.8, 0.1	<0.4,0.4>
$\mathfrak{D}_1$	A^-	<0.0,0.8>	<0.4,0.5>	<0.2,0.8>	<0.1,0.7>	<0.1,0.8>
	A^+	<0.4,0.0>	<0.7,0.0>	<0.8,0.1>	<0.8,0.0>	<0.4,0.0>
$\mathfrak{D}_2$	A^-	<0.1,0.7>	<0.0,0.9>	<0.2,0.7>	<0.1,0.8>	<0.1,0.7>
	A^+	<0.7,0.2>	<0.6, 02>	<0.7,0.2>	<0.7,0.2>	<0.6,0.0>
$\mathfrak{D}_3$	A^-	<0.2,0.8>	<0.2,0.8>	<0.3,0.6>	<0.1,0.7>	<0.2,0.8>

Step 4: Calculate the separation distance of each competitive alternative from the ideal and non- ideal solution

Separation measures $D(\mathcal{A}_i, A^+)$, $D(\mathcal{A}_i, A^-)$ and the weighted exponential divergence measure proposed in equation (7) of each alternative from FPIS and FNIS have been calculated using formulae (10) and (11) and presented in table 9.

$$D(\mathcal{A}_i, A^+) = \frac{2}{n}\sum_{i=1}^{n}\omega_i\left[\left|\delta_P^2(x_i) - \delta_Q^{2+}(x_i)\right|.\,2^{-\left|\delta_P^2(x_i)-\delta_Q^{2+}(x_i)\right|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^{2+}(x_i)\right|.\,2^{-\left|\zeta_P^2(x_i)-\zeta_Q^{2+}(x_i)\right|-1}\right] \quad (10)$$

$$D(\mathcal{A}_i, A^-) = \frac{2}{n}\sum_{i=1}^{n}\omega_i\left[\left|\delta_P^2(x_i) - \delta_Q^{2-}(x_i)\right|.\,2^{-\left|\delta_P^2(x_i)-\delta_Q^{2-}(x_i)\right|-1} + \left|\zeta_P^2(x_i) - \zeta_Q^{2-}(x_i)\right|.\,2^{-\left|\zeta_P^2(x_i)-\zeta_Q^{2-}(x_i)\right|-1}\right] \quad (11)$$

Table 9. Separation measures for ideal solutions *w.r.t.* each decision maker

Alternatives	$Ɗ_1$		$Ɗ_2$		$Ɗ_3$	
	$D^1(A_i, A^+)$	$D^1(A_i, A^-)$	$D^2(A_i, A^+)$	$D^1(A_i, A^+)$	$D^1(A_i, A^-)$	$D^2(A_i, A^+)$
$\mathcal{A}_1$	0.03861	0.08584	0.03659	0.10277	0.06359	0.05815
$\mathcal{A}_2$	0.09158	0.03120	0.08431	0.07717	0.10660	0.03200
$\mathcal{A}_3$	0.05103	0.08746	0.11772	0.01963	0.05403	0.06523
$\mathcal{A}_4$	0.03305	0.09852	0.10238	0.04890	0.04483	0.06909
$\mathcal{A}_5$	0.05061	0.07748	0.05728	0.09883	0.06889	0.06916

Step 5: Measure the relative closeness of each location to the ideal solution and rank the preference order

For each competitive alternative the relative closeness of the potential model with respect to the ideal solution is computed. Relative closeness coefficient with respect to each decision maker can be found using the formula

$$R_i = \frac{D(A_i, A^-)}{D(A_i, A^+) + D(A_i, A^-)} \quad (12)$$

where $0 \leq R_{\mathrm{i}} \leq 1, i = 1,2, \dots, m$

The value of R_i signifies that higher the value of the relative closeness, the higher the ranking order and hence the better the performance of the alternative. Ranking of the preference in descending order thus allows relatively better performances to be compared. The ranking results obtained by Pythagorean fuzzy TOPSIS approach is demonstrated in table 10.

Table 10. Ranking Results Obtained from TOPSIS Approach

Alternatives	$Ɗ_1$		$Ɗ_2$		$Ɗ_3$	
	R_1	*Ranking*	R_2	*Ranking*	R_3	*Ranking*
$\mathcal{A}_1$	0.6897	2	0.7374	1	0.4776	4
$\mathcal{A}_2$	0.2541	5	0.4779	3	0.2308	5
$\mathcal{A}_3$	0.6315	3	0.1428	5	0.5469	2
$\mathcal{A}_4$	0.74882	1	0.3232	4	0.6064	1
$\mathcal{A}_5$	0.6048	4	0.6330	2	0.5009	3

5. Discussion

Table 10 shows the ranking of the 5 considered smart phones according to the three decision makers. For decision maker 1, D^1, Model 4 ($\mathcal{A}_4$) is the best smartphone. Same is the case for decision makers three, D^3. On the other hand, Model 1 ($\mathcal{A}_1$) is the best choice for decision maker D^2. Further, to validate this result, Sensitivity analysis will be carried out in the next section,

5.1. Sensitivity Analysis

If decision makers find distinct ranking for the alternatives, the overall findings of the best alternatives remain unclear. To overcome the ambiguity about the best alternatives with respect to the decision-makers, we aggregate the ideal distance measurement values of every decision-maker different values of the experts are aggregated by assigning a priority value value $\rho = (\rho_1, \rho_2, \dots, \rho_s)^T$ to each expert such that $\rho_s > 0$ and $\sum_{k=1}^{s} \rho_k = 1$.

The distance measure of each expert is aggregated by using these weight vectors and the overall measurement values of the alternatives are obtained which can be depicted in table 11 as

$$\vartheta_i^+ = \sum_{k=1}^{s} \rho_k C_{ij}^+ \tag{13}$$

$$\vartheta_i^- = \sum_{k=1}^{s} \rho_k C_{ij}^- \tag{14}$$

Also, $\Re_i = \frac{\vartheta_i^-}{\vartheta_i^+ + \vartheta_i^-}$ (15)

where $0 \le \Re_i \le 1, i = 1, 2, \dots, 5$

Table 11. Aggregated closeness coefficient and ranking for each smartphone

Case 1: $\rho_1 = 0.45, \rho_2 = 0.35, \rho_3 = 0.20$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6677	1	
$\mathcal{A}_2$	0.3401	5	
$\mathcal{A}_3$	0.4415	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5577	3	
$\mathcal{A}_5$	0.5953	2	
Case 2: $\rho_1 = 0.35, \rho_2 = 0.27, \rho_3 = 0.37$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6268	1	
$\mathcal{A}_2$	0.3154	5	
$\mathcal{A}_3$	0.4637	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5679	3	
$\mathcal{A}_5$	0.5743	2	
Case 3: $\rho_1 = 0.38, \rho_2 = 0.33, \rho_3 = 0.29$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6485	1	
$\mathcal{A}_2$	0.3325	5	
$\mathcal{A}_3$	0.4423	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5536	3	
$\mathcal{A}_5$	0.5855	2	
Case 4: $\rho_1 = 0.40, \rho_2 = 0.30, \rho_3 = 0.30$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6448	1	
$\mathcal{A}_2$	0.3250	5	
$\mathcal{A}_3$	0.4565	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5659	3	
$\mathcal{A}_5$	0.5834	2	

$Case\ 5$: $\rho_1 = 0.29, \rho_2 = 0.25, \rho_3 = 0.46$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6092	1	
$\mathcal{A}_2$	0.3081	5	
$\mathcal{A}_3$	0.4659	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5653	3	
$\mathcal{A}_5$	0.5655	2	

$Case\ 6$: $\rho_1 = 0.29, \rho_2 = 0.25, \rho_3 = 0.46$			
Alternatives	$\Re_i$	*Ranking*	*Selected smartphone*
$\mathcal{A}_1$	0.6626	1	
$\mathcal{A}_2$	0.3347	5	
$\mathcal{A}_3$	0.4495	4	$\mathcal{A}_1$
$\mathcal{A}_4$	0.5639	3	
$\mathcal{A}_5$	0.5926	2	

Sensitivity Analysis concluded that by assigning different priorities to the opinions of decision makers, the result of the proposed method remains the same, as $\mathcal{A}_1$ came out to be the Best-Model in all the cases, thereby substantiating the validity and reliability of the proposed method.

6. Comparative Study

To demonstrate the dominance of the proposed divergence measure, a comparison between the proposed weighted exponential divergence measure and the existing measures is conducted based on the numerical cases suggested. Table 12 represents a comprehensive evaluation of the divergence measures for PFSs.

The numerical data in table 12 have been analysed, and it has been discovered that the results produced using our suggested divergence measure given in equation 8 are like those obtained using existing measures. As a result, the accompanying table demonstrates that the proposed divergence measure is consistent across all approaches, as the best alternative remains the same.

Table 12. Comparison of existing with the proposed divergence measures

Measure	Ranking
Peng et al. (2017)	$\mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_5 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Ejegwa (2018) Measure I	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Ejegwa (2018) Measure II	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Ejegwa (2018) Measure III	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Zhang et al. (2019) Measure I	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Zhang et al. (2019) Measure II	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Zhang et al. (2019) Measure III	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$
Zhang et al. (2019) Measure IV	$\mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$

It can be determined by studying the above findings that there are differences in ranks when different scenarios are applied, indicating that the model is sensitive to changes. It is observed that $\mathcal{A}_1$ is the best alternative. In accordance with the findings of Bobar et al. (2020), Spearman's rank correlation is used to find the correlation of attributes.

When written in mathematical form, Spearman's Coefficient of correlation is denoted by $\mathcal{R}$ and is defined by

$$\mathcal{R} = 1 - \frac{6\sum_{i=1}^{n} \check{D}_i^2}{n(n^2-1)} \tag{16}$$

where $\check{D}_i$= difference in ranks of the "ith" element: n = number of observations
$\mathcal{R}$ = value of correlation coefficient

The value of $\mathcal{R}$ will always lies between -1 and 1. If $\mathcal{R} = 1$, there is a perfectly positive correlation; If $\mathcal{R} = -1$, then the ranks are exactly opposite. However, if $\mathcal{R} = 0$, then the ranks are uncorrelated. Spearman's rank correlation among existing and proposed measures is shown in table 13 as

Table 13. Spearman's rank correlation for various existing and proposed measures

	Peng et al. (2017)	Ejegwa (2018) Measure I	Ejegwa (2018) Measure II	Ejegwa (2018) Measure III	Zhang et al. (2019) Measure I	Zhang et al. (2019) Measure II	Zhang et al. (2019) Measure III	Zhang et al. (2019) Measure III	Zhang et al. (2019) Measure IV	Proposed Measure
Peng et al. (2017)	1									
Ejegwa (2018) Measure I	0.9	1								
Ejegwa,(2018) Measure II	0.9	1	1							
Ejegwa (2018) Measure III	0.9	1	1	1						
Zhang et al. (2019) Measure I	0.9	1	1	1	1					
Zhang et al. (2019) Measure II	0.9	1	1	1	1	1				
Zhang et al. (2019) Measure III	0.9	1	1	1	1	1	1			
Zhang et al. (2019) Measure III	0.9	1	1	1	1	1	1	1		
Zhang et al. (2019) Measure IV	0.9	1	1	1	1	1	1	1	1	
Proposed Measure	0.9	1	1	1	1	1	1	1	1	1

The table of Spearman's coefficient of correlation values shows that there is perfectly positive correlation in almost all the cases.
The graphical representation (Figure 2) is also shown for better understanding of the selection procedure.

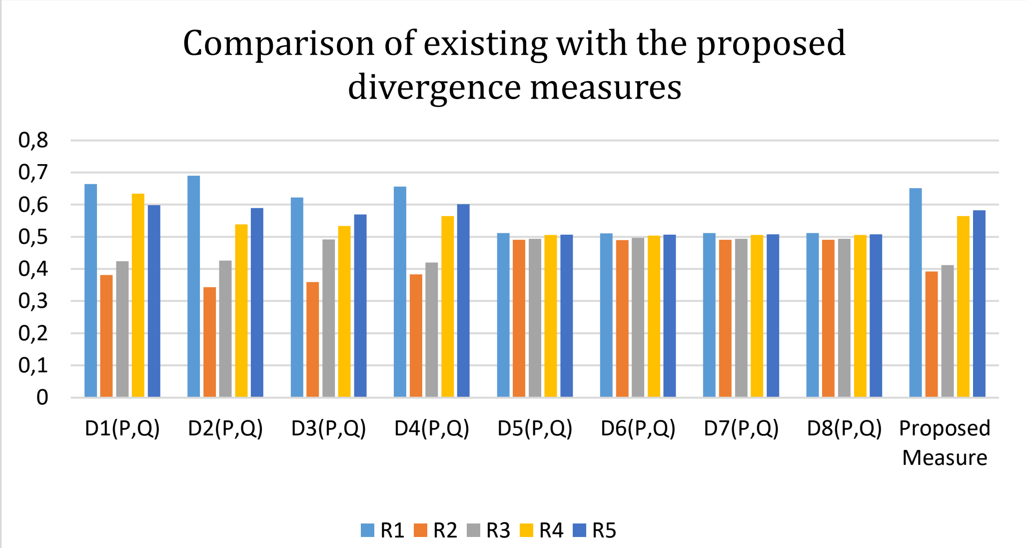

Figure 2. Comparison of existing with the projected divergence measure

7. Conclusion

The paper offers new exponential divergence measures which comply with the conventional parameters of PFSs. The credibility of the proposed divergence measures through numerical computations has been confirmed as well. Further, these divergence measures have been employed to the application of MADM problem for the selection of smartphones. This analysis depicts an extension of TOPSIS methodology under Pythagorean fuzzy sets (PFSs) environment. The technique for order preference by similarity to ideal solutions (TOPSIS) notable and powerful technique for multi attribute decision making (MADM) issues. The goal of this investigation is to broaden TOPSIS to handle MADM problems under PFSs. However, in this TOPSIS approach, sometimes there could be severe loss of data and misleading results in the fuzzy environment. To overcome this, a sensitivity analysis has been done for better reliability and accuracy of the decision. A case study is taken to rank five leading smartphones based on five criteria using the proposed divergence measures. These weighted divergence measures can be applied to complex decision making and risk analysis in the future. Furthermore, criteria weights can be chosen using Entropy and sensitivity analysis of the obtained results with the results obtained by the basic classical methods TOPSIS, fuzzy TOPSIS methods, and Intuitionistic fuzzy TOPSIS can be done.

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