Decision Making: Applications in Management and Engineering
Vol. 5, Issue 2, 2022, pp. 30-45.
ISSN: 2560-6018
eISSN: 2620-0104
DOI: https://doi.org/10.31181/dmame0305102022d

THE ENERGY OF ROUGH NEUTROSOPHIC MATRIX AND ITS APPLICATION TO MCDM PROBLEM FOR SELECTING THE BEST BUILDING CONSTRUCTION SITE

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Received: 10 August 2022;
Accepted: 26 September 2022;
Available online: 5 October 2022.

Original scientific paper

Abstract: *An approach to data processing for relational databases is called a rough set theory. It is an interesting area of uncertainty mathematics that is mainly related to fuzzy theory. Rough set theory and neutrosophic set theory can be joined to create a powerful tool for dealing with indeterminacy. Neutrosophic matrices help decision-makers deal with multi-criteria decision-making by providing them with more useful and practical when we apply the concept of matrix energy. In this paper, we defined a Rough neutrosophic matrix and its energy. Some propositions, lower and upper limits of the rough neutrosophic matrix's energy were derived. The proposed energy of the rough neutrosophic matrix was applied in multi-criteria decision-making problems. The problem is to select the best place for constructing the school building. Applying the energy method to the MCDM problem became more relatable and produced good results.*

Keywords: *Rough Set, Rough Neutrosophic Set, Rough Neutrosophic Matrix, Energy of Rough Neutrosophic Matrix, Multi-Criteria Decision Making (MCDM).*

1. Introduction

Fuzzy sets, fuzzy membership functions, and fuzzy logic were first introduced (Zadeh, 1965). Fuzzy Matrix Theory, which focused on the convergence of fuzzy matrices' powers, was first presented by Thomason (1977). It can be applied in several circumstances. It is commonly known that the matrix representation offers an additional advantage in resolving the issue. Intuitionistic fuzzy matrices were first introduced (Pal et al., 2002). It is difficult to determine the value of membership or non-membership as a point, though. The Neutrosophic set was first introduced (Smarandache, 1998). He put out the concepts of Neutrosophic Set, Probability, and

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Logic to specifically address the issue of indeterminacy. He also created hesitant and dual hesitant neutrosophic sets, single and interval valued neutrosophic sets, and multi-valued neutrosophic sets. After that the introduction of fuzzy relational maps and neutrosophic relational maps was presented (Kandasamy & Smarandache, 2004). In this, they included square neutrosophic matrices. The neutrosophic matrix and associated algebraic operations were created (Dhar et al., 2014).

Christi DiStefano and colleagues introduced the idea of matrix energy in 2009. They devised the equation for the matrix's energy. A generalization of the energy of a graph is the energy of a matrix. A paper titled Energy of Matrixes was Proposed (Bravo et al., 2017). They produced a number of theorems on matrix energy as well as upper and lower bounds. The notion of matrix energy does not hold true in a neutrosophic setting or for MCDM problems, however, the notion of graph energy will gain in popularity. We, therefore, examine energy in neutrosophic matrices and how it might be used in MCDM in this paper.

The idea of a rough set was first introduced by Pawlak (1982). Its foundation is the approximation of sets by a pair of sets known as the lower and upper approximations of a set. In this instance, the equivalence relation is the basis for those approximations. Then he compares the fuzzy set with the rough set concept (Pawlak, 1985). Then the rough set techniques for incomplete information systems were presented, along with fuzzy rough sets (Kryszkiewicz, 1998). The Rough Intuitionistic fuzzy set was proposed (Rizvi et al., 2002) They defined the Rough Intuitionistic fuzzy set and its properties. The Generalized fuzzy rough sets were introduced (Wu et al., 2003). This paper studies fuzzy rough sets using both constructive and axiomatic approaches. Then the rough set and fuzzy rough set on interval-valued fuzzy was proposed (Gong et al., 2008). Both axiomatic and constructive methods to develop a complete framework for the study of interval type-2 rough fuzzy sets are used (Zhang, 2012). A concept of the Rough Fuzzy set model for a set-valued ordered fuzzy decision system was presented (Bao et al., 2014). In order to create decision rules for long-term forecasting of air passengers suggested a novel hybrid method based on rough set theory (Sharma et al., 2018). Then they used a combination technique to evaluate India's sugarcane production based on the rough set approach (Sharma et al., 2021). They also provide a basic decision-making process based on a set theory for assessing the performance of Delhi hotels (Sharma et al., 2022). They develop a rough set theory to offer a set of decision rules and significant feature sets.

Rough set theory and neutrosophic set theory will both be useful methods for handling incomplete, ambiguous, uncertain, and incorrect data. (Broumi et al., 2014) introduced the idea of the Rough Neutrosophic Set. They outlined the rough neutrosophic sets and their operations in this study. Then they proposed the Interval Valued Neutrosophic Rough Set (Broumi et al., 2015). A rough grey relational analysis-based strategy for neutrosophic multi-attribute decision-making is illustrated (Mondal & Pramanik, 2015). In this work, the rough neutrosophic decision matrix is defined, and an MCDM issue is solved using this matrix. A number of authors offered various ideas for a rough neutrosophic field (Alias et al., 2017; Yang et al., 2017; Pramanik et al., 2017). They studied MCDM in rough neutrosophic sets with coefficient correlation, rough single-valued neutrosophic sets, and rough neutrosophic multisets. On novel multi-granulation neutrosophic rough set on a single value and its uses was discussed (Bo et al., 2018). Rough Neutrosophic Set is used in medical diagnosis (Samuel & Narmadhagnanam, 2018). This paper discusses the use of medical diagnostics to identify the patient's health. An article with the title medical diagnosis focused on single-valued neutrosophic uncertain rough multisets over two universes was published in the same year (Zhang et al., 2018). Rough Neutrosophic Sets Pi-Distance for Medical Diagnosis was presented (Samuel & Narmadhagnanam, 2019). The objective of the study is to establish a causal relationship between the illness and the patient's symptoms and to examine the patient's state using a rough neutrosophic set. The notions of the neutrosophic soft set with rough set theory (Das et al., 2021), neutrosophic single-valued rough sets, and including topology (Jin et al., 2021) will be further developed. The rough set is important in every field of the neutrosophic environment as result.

Multi-Criteria Decision Making (MCDM) is a key and quickly developing subject in operations research. Indeterminacy should be handled in the modeling approach of challenges since MCDM problems are well addressed in fuzzy. The development of the MCDM field in a fuzzy environment led to the proposal of the Neutrosophic Fuzzy MCDM, which was used in numerous methods (Otay, 2022; Wang & Zhang, 2022). There are some more different methods in MCDM to select the best alternatives. Recently, several researchers work on many types of methods (Gorcun et al., 2021; Arora & Naithani, 2022). In this study, we added a new step to the method for resolving MCDM issues with a rough neutrosophic matrix by determining its energy. In section 2, The fundamental definitions are provided. In section 3, we introduced the energy of the rough neutrosophic matrix together with its hypotheses, and upper and lower bounds. A new strategy named the Rough Neutrosophic Energy Method was introduced in section 4 and described in detail. The numerical example of the suggested method was resolved in section 5. Then, a conclusion was given.

2. Preliminaries

Definition 2.1. Rough set (Pawlak, 1982)

Let U be the universal set and R be an equivalence relation on U (this is called an indiscernibility relation). The collection of all equivalence classes of U under R is defined as $A = U/R$, which is called an approximation space.

Let $X \subseteq U$ be a subset of U. We define lower and upper approximation of X in A, denoted $\underline{A}(X)$ and $\overline{A}(X)$ respectively, as follows

$$\underline{A}(X) = \{a \in U : [a]_R \subseteq X\}$$

$$\overline{A}(X) = \{a \in U : [a]_R \cap X \neq \emptyset\}$$

where $[a]_R$ denotes the equivalence class of R containing an element a.

The pair $A(X) = (\underline{A}(X), \overline{A}(X))$ is called the rough set of X in A.

Definition 2.2. Neutrosophic Set (Smarandache, 1998)

Let U be the universal set and every element $a \subseteq U$ has a degree of True, Indeterminacy, False membership in neutrosophic set. It is denoted by S. Then it can be written as

$$S = \{ \langle a, T_{S(a)}, I_{S(a)}, F_{S(a)} \rangle : a \in U\}$$

where, $0 \leq T_{S(a)} + I_{S(a)} + F_{S(a)} \leq 3$ and Truth Membership function $T_S: U \rightarrow [0,1]$, Indeterminacy Membership function $I_S: U \rightarrow [0,1]$, False Membership function $F_S: U \rightarrow [0,1]$.

Definition 2.3. Rough Neutrosophic Set (Broumi et al., 2014)

Let U be the universal set and every element $a \in U$. Let R be an equivalence relation on U and S be the neutrosophic set in U with truth membership function T_S, indeterminacy function I_S and false membership function F_S. The lower and upper approximations of S in U/R is denoted by $\underline{N}(X)$ and $\overline{N}(X)$ and they are defined as follows,

$$\underline{N}(S) = \{\langle a, T_{\underline{N}(S)}(a), I_{\underline{N}(S)}(a), F_{\underline{N}(S)}(a)\rangle : b \in [a]_R, a \in U\}$$

$$\overline{N}(S) = \{\langle a, T_{\overline{N}(S)}(a), I_{\overline{N}(S)}(a), F_{\overline{N}(S)}(a)\rangle : b \in [a]_R, a \in U\}$$

where,

$$T_{\underline{N}(S)}(a) = \bigwedge_{b \in [a]_R} T_S(b) \qquad T_{\overline{N}(S)}(a) = \bigvee_{b \in [a]_R} T_S(b)$$

$$I_{\underline{N}(S)}(a) = \bigvee_{b \in [a]_R} I_S(b) \qquad I_{\overline{N}(S)}(a) = \bigwedge_{b \in [a]_R} I_S(b)$$

$$F_{\underline{N}(S)}(a) = \bigvee_{b \in [a]_R} F_S(b) \qquad F_{\overline{N}(S)}(a) = \bigwedge_{b \in [a]_R} F_S(b)$$

where, $0 \le T_{\underline{N}(S)}(a) + I_{\underline{N}(S)}(a) + F_{\underline{N}(S)}(a) \le 3$ and $0 \le T_{\overline{N}(S)}(a) + I_{\overline{N}(S)}(a) + F_{\overline{N}(S)}(a) \le 3$. Where, $\vee$ means 'max' and $\wedge$ means 'min' and $T_S(a), I_S(a), F_S(a)$ are truth, indeterminacy, false membership function of a on S. Therefore $\underline{N}(S)$ and $\overline{N}(S)$ are two neutrosophic sets in U. The pair $(\underline{N}(S), \overline{N}(S))$ is called the Rough Neutrosophic set in U/R.

If $\underline{N}(S) = \overline{N}(S)$ for any $a \in U$, then S is called definable neutrosophic set.

Definition 2.4. Energy of Matrix (Bravo et al. 2017)

Let $M_{n(\mathbb{C})}$ denote the space of $n \times n$ matrices with entries in $\mathbb{C}$ and P be a matrix in $M_{n(\mathbb{C})}$. We define the energy of A as

$$E(P) = \sum_{i=1}^{n} |\lambda_i - \mu|$$

where, $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigenvalues of P and μ is the mean of eigenvalues. If $\mu = 0$ or P is the adjacency matrix of a graph G then E(P) is precisely the energy of the graph G.

Definition 2.5. Energy of Neutrosophic Matrix

Let P(N) be the Neutrosophic matrix with the order of $n \times n$ (square matrix). It can be expressed as three matrices, the first matrix contains the entries a_{ij} as truth membership values, the second contains the entries b_{ij} as indeterminacy membership values and the third matrix contains the entries c_{ij} as false membership values.

It is denoted as $P(N) = \langle P(T_{ij}), P(I_{ij}), P(F_{ij})\rangle_{n\times n}$ and $a_{ij} \in P(T_{ij})_{n\times n}, b_{ij} \in P(I_{ij})_{n\times n}$ *and* $c_{ij} \in P(F_{ij})_{n\times n}$

The energy of a neutrosophic matrix is defined as

$$E[P(N)] = \langle E[P(T_{ij})], E[P(I_{ij})], E[P(F_{ij})] \rangle$$

$$= \langle \sum_{i=1}^{n} |\lambda_i - \mu|, \sum_{i=1}^{n} |\zeta_i - \mu|, \sum_{i=1}^{n} |\eta_i - \mu| \rangle$$

where, λ_i, ζ_i and η_i, $(i = 1,2, \ldots n)$ are the eigenvalues of Truth, Indeterminacy, and False membership values respectively and μ_λ, μ_ζ, and μ_η are the mean values of λ_i, ζ_i and η_i respectively.

3. Energy of Rough Neutrosophic Matrix

Definition 3.1. Energy of Rough Neutrosophic Matrix

Let $D(N) = \langle D(\underline{N}_{ij}(S)), D(\overline{N}_{ij}(S)) \rangle$ be the Rough Neutrosophic matrix with the order $n \times n$. where, $D(\underline{N}_{ij}(S))$ and $D(\overline{N}_{ij}(S))$ are a lower and upper approximation of the neutrosophic set S. The rough neutrosophic matrix can be expressed as 6 matrices, first 3 matrices are under lower approximation which contains the elements $\underline{a}_{ij}$, $\underline{b}_{ij}$, $\underline{c}_{ij}$, another 3 matrices are under upper approximation which contains the elements $\overline{a}_{ij}, \overline{b}_{ij}, \overline{c}_{ij}$. where, $\underline{a}_{ij}, \overline{a}_{ij}$ are truth membership values, $\underline{b}_{ij}, \overline{b}_{ij}$ are indeterminacy membership values and $\underline{c}_{ij}, \overline{c}_{ij}$ are false membership values. which is denoted as, $D(N) = \langle D(\underline{N}_{ij}(S)), D(\overline{N}_{ij}(S)) \rangle$

$$= \langle (D\left(\underline{T}_{ij}(S)\right), D\left(\underline{I}_{ij}(S)\right), D\left(\underline{F}_{ij}(S)\right)), (D\left(\overline{T}_{ij}(S)\right), D\left(\overline{I}_{ij}(S)\right), D\left(\overline{F}_{ij}(S)\right)) \rangle$$

where the elements, $\underline{a}_{ij} \in D\left(\underline{T}_{ij}(S)\right), \underline{b}_{ij} \in D\left(\underline{I}_{ij}(S)\right), c_{ij} \in D\left(\underline{F}_{ij}(S)\right)$, $\overline{a}_{ij} \in D\left(\overline{T}_{ij}(S)\right)$, $\overline{b}_{ij} \in D\left(\overline{I}_{ij}(S)\right), \overline{c}_{ij} \in D\left(\overline{F}_{ij}(S)\right)$.

Then the energy of Rough Neutrosophic matrix defined as

$$E[D(N)] = \langle (E[D\left(\underline{T}_{ij}(S)\right)], E[D\left(\underline{I}_{ij}(S)\right)], E[D\left(\underline{F}_{ij}(S)\right)]), \\ (E[D\left(\overline{T}_{ij}(S)\right)], E[D\left(\overline{I}_{ij}(S)\right)], E[D\left(\overline{F}_{ij}(S)\right)]) \rangle$$

$$E[D(N)] = \langle \begin{matrix} \left(\sum_{i=1}^{n} |\underline{\lambda}_i - \mu_{\underline{\lambda}}|, \sum_{i=1}^{n} |\underline{\zeta}_i - \mu_{\underline{\zeta}}|, \sum_{i=1}^{n} |\underline{\eta}_i - \mu_{\underline{\eta}}| \right), \\ \left(\sum_{i=1}^{n} |\overline{\lambda}_i - \mu_{\overline{\lambda}}|, \sum_{i=1}^{n} |\overline{\zeta}_i - \mu_{\overline{\zeta}}|, \sum_{i=1}^{n} |\overline{\eta}_i - \mu_{\overline{\eta}}| \right) \end{matrix} \rangle$$

where, $\underline{\lambda}_i$, $\underline{\zeta}_i$, $\underline{\eta}_i$ are the eigenvalues of truth, indeterminacy, and false values of lower approximation matrices and $\overline{\lambda}_i$, $\overline{\zeta}_i$, $\overline{\eta}_i$ are the eigenvalues of truth, indeterminacy, and false values of upper approximation matrices. $\mu_{\underline{\lambda}}, \mu_{\underline{\zeta}}, \mu_{\underline{\eta}}, \mu_{\overline{\lambda}}, \mu_{\overline{\zeta}}$ and $\mu_{\overline{\eta}}$ are mean values of the eigen values $\underline{\lambda}_i, \underline{\zeta}_i, \underline{\eta}_i, \overline{\lambda}_i, \overline{\zeta}_i$ and $\overline{\eta}_i$ respectively.

Example:

Let D be the Rough Neutrosophic Matrix with the order of 3×3.

$$D=\begin{bmatrix}\langle(.8,.6,.7),(.9,.3,.4)\rangle & \langle(.5,.7,.7),(.6,.4,.5)\rangle & \langle(.1,.4,.5),(.5,.2,.3)\rangle\\ \langle(.4,.6,.5),(.7,.3,.2)\rangle & \langle(.6,.6,.7),(.8,.5,.3)\rangle & \langle(.5,.7,.8),(.6,.1,.2)\rangle\\ \langle(.3,.5,.6),(.7,.2,.1)\rangle & \langle(.2,.7,.8),(.9,.2,.3)\rangle & \langle(.4,.6,.9),(.7,.4,.3)\rangle\end{bmatrix}_{n\times n}$$

D can be expressed as 6 matrices.

$$D(\underline{T}_{ij})=\begin{pmatrix}0.8 & 0.5 & 0.1\\ 0.4 & 0.6 & 0.5\\ 0.3 & 0.2 & 0.4\end{pmatrix}\ D(\underline{I}_{ij})=\begin{pmatrix}0.6 & 0.7 & 0.4\\ 0.6 & 0.6 & 0.7\\ 0.5 & 0.7 & 0.6\end{pmatrix}\ D(\underline{F}_{ij})=\begin{pmatrix}0.7 & 0.7 & 0.5\\ 0.5 & 0.7 & 0.8\\ 0.6 & 0.8 & 0.9\end{pmatrix}$$

$$D(\overline{T}_{ij})=\begin{pmatrix}0.9 & 0.6 & 0.5\\ 0.7 & 0.8 & 0.6\\ 0.7 & 0.9 & 0.7\end{pmatrix}\ \overline{I}_{ij}(S)=\begin{pmatrix}0.3 & 0.4 & 0.2\\ 0.3 & 0.5 & 0.1\\ 0.2 & 0.2 & 0.4\end{pmatrix}\ \overline{F}_{ij}(S)=\begin{pmatrix}0.4 & 0.5 & 0.3\\ 0.2 & 0.3 & 0.2\\ 0.1 & 0.3 & 0.3\end{pmatrix}$$

Energy of D matrix, $E(D)=\langle(1.4749, 2.4105, 2.6262),(2.6387, 0.9544, 1.0003)\rangle$

Theorem 3.3.

Let D(N) be the Rough neutrosophic matrix. If $\underline{\lambda}_i, \underline{\zeta}_i, \underline{\eta}_i, \overline{\lambda}_i, \overline{\zeta}_i$ and $\overline{\eta}_i$, $(i=1,2,\dots n)$ are the eigenvalues of lower approximation of Truth $D(\underline{T}_{ij})$, Indeterminacy $D(\underline{I}_{ij})$, and False $D(\underline{F}_{ij})$ and upper approximation of Truth $D(\overline{T}_{ij})$, Indeterminacy $D(\overline{I}_{ij})$, and False $D(\overline{F}_{ij})$ membership values respectively.

$$1)\ \sum_{i=1}^{n}|\underline{\lambda}_i-\mu_{\underline{\lambda}}|=\sum_{i=1}^{n}|\underline{a}_{ii}-\mu_{\underline{\lambda}}| = \sum_{i=1}^{n}|\overline{\lambda}_i-\mu_{\overline{\lambda}}| = \sum_{i=1}^{n}|\overline{a}_{ii}-\mu_{\overline{\lambda}}|=0$$

$$\sum_{i=1}^{n}|\underline{\zeta}_i-\mu_{\underline{\zeta}}|=\sum_{i=1}^{n}|\underline{b}_{ii}-\mu_{\underline{\lambda}}| = \sum_{i=1}^{n}|\overline{\zeta}_i-\mu_{\overline{\zeta}}| = \sum_{i=1}^{n}|\overline{b}_{ii}-\mu_{\overline{\zeta}}|=0$$

$$\sum_{i=1}^{n}|\underline{\eta}_i-\mu_{\underline{\eta}}|=\sum_{i=1}^{n}|\underline{c}_{ii}-\mu_{\underline{\eta}}| = \sum_{i=1}^{n}|\overline{\eta}_i-\mu_{\overline{\eta}}| = \sum_{i=1}^{n}|\overline{c}_{ii}-\mu_{\overline{\eta}}|=0$$

$$2)\ \sum_{i=1}^{n}\left(\underline{\lambda}_i-\mu_{\underline{\lambda}}\right)^2=\sum_{i=1}^{n}\underline{a}_{ii}^{\,2}+2\sum_{1\le i<j\le n}\underline{a}_{ij}\underline{a}_{ji}-n\,\mu_{\underline{\lambda}}^{\,2},$$

$$\sum_{i=1}^{n}\left(\overline{\lambda}_i-\mu_{\overline{\lambda}}\right)^2=\sum_{i=1}^{n}\overline{a}_{ii}^{\,2}+2\sum_{1\le i<j\le n}\overline{a}_{ij}\overline{a}_{ji}-n\,\mu_{\overline{\lambda}}^{\,2}$$

$$\sum_{i=1}^{n}\left(\underline{\zeta}_i-\mu_{\underline{\zeta}}\right)^2=\sum_{i=1}^{n}\underline{a}_{ii}^{\,2}+2\sum_{1\le i<j\le n}\underline{a}_{ij}\underline{a}_{ji}-n\,\mu_{\underline{\zeta}}^{\,2},$$

$$\sum_{i=1}^{n}\left(\overline{\zeta}_i-\mu_{\overline{\zeta}}\right)^2=\sum_{i=1}^{n}\overline{a}_{ii}^{\,2}+2\sum_{1\le i<j\le n}\overline{a}_{ij}\overline{a}_{ji}-n\,\mu_{\overline{\zeta}}^{\,2},$$

$$\sum_{i=1}^{n}\left(\underline{\eta}_i-\mu_{\underline{\eta}}\right)^2=\sum_{i=1}^{n}\underline{a}_{ii}^{\,2}+2\sum_{1\le i<j\le n}\underline{a}_{ij}\underline{a}_{ji}-n\,\mu_{\underline{\eta}}^{\,2},$$

$$\sum_{i=1}^{n}(\overline{\eta}_i-\mu_{\overline{\eta}})^2=\sum_{i=1}^{n}\overline{a}_{ii}{}^2+2\sum_{1\le i<j\le n}\overline{a}_{ij}\overline{a}_{ji}-n\,\mu_{\overline{\eta}}{}^2$$

Theorem 3.4.

Let $D(N)=\langle\left(D(\underline{T}_{ij}),D(\underline{I}_{ij}),D(\underline{F}_{ij})\right),\left(D(\overline{T}_{ij}),D(\overline{I}_{ij}),D(\overline{F}_{ij})\right)\rangle$ be the Rough neutrosophic matrix. Then the lower and upper bound of each energy is as follows

$$i)\ \sqrt{\left(\sum_{i=1}^{n}|\underline{\lambda}_i-\mu_{\underline{\lambda}}|\right)^2-2\sum_{1\le i<j\le n}|\underline{\lambda}_i-\mu_{\underline{\lambda}}||\underline{\lambda}_j-\mu_{\underline{\lambda}}|+n(n-1)[|D-\mu_{\underline{\lambda}}|]^{\frac{2}{n}}}\le E\left(D(\underline{T}_{ij})\right)$$
$$\le\sqrt{2\left[\left(\sum_{i=1}^{n}|\underline{\lambda}_i-\mu_{\underline{\lambda}}|\right)^2-2\sum_{1\le i<j\le n}|\underline{\lambda}_i-\mu_{\underline{\lambda}}||\underline{\lambda}_j-\mu_{\underline{\lambda}}|\right]}$$

$$ii)\ \sqrt{\left(\sum_{i=1}^{n}\left|\underline{\zeta}_i-\mu_{\underline{\zeta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\underline{\zeta}_i-\mu_{\underline{\zeta}}\right|\left|\underline{\zeta}_j-\mu_{\underline{\zeta}}\right|+n(n-1)\left[\left|D-\mu_{\underline{\zeta}}\right|\right]^{\frac{2}{n}}}$$
$$\le E\left(D(\underline{I}_{ij})\right)\le\sqrt{2\left[\left(\sum_{i=1}^{n}\left|\underline{\zeta}_i-\mu_{\underline{\zeta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\underline{\zeta}_i-\mu_{\underline{\zeta}}\right|\left|\underline{\zeta}_j-\mu_{\underline{\zeta}}\right|\right]}$$

$$iii)\ \sqrt{\left(\sum_{i=1}^{n}\left|\underline{\eta}_i-\mu_{\underline{\eta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\underline{\eta}_i-\mu_{\underline{\eta}}\right|\left|\underline{\eta}_j-\mu_{\underline{\eta}}\right|+n(n-1)\left[\left|D-\mu_{\underline{\eta}}\right|\right]^{\frac{2}{n}}}$$
$$\le E\left(D(\underline{F}_{ij})\right)\le\sqrt{2\left[\left(\sum_{i=1}^{n}\left|\underline{\eta}_i-\mu_{\underline{\eta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\underline{\eta}_i-\mu_{\underline{\eta}}\right|\left|\underline{\eta}_j-\mu_{\underline{\eta}}\right|\right]}$$

$$iv)\ \sqrt{\left(\sum_{i=1}^{n}|\overline{\lambda}_i-\mu_{\overline{\lambda}}|\right)^2-2\sum_{1\le i<j\le n}|\overline{\lambda}_i-\mu_{\overline{\lambda}}||\overline{\lambda}_j-\mu_{\overline{\lambda}}|+n(n-1)[|D-\mu_{\underline{\lambda}}|]^{\frac{2}{n}}}$$
$$\le E\left(D(\underline{T}_{ij})\right)\le\sqrt{2\left[\left(\sum_{i=1}^{n}|\overline{\lambda}_i-\mu_{\overline{\lambda}}|\right)^2-2\sum_{1\le i<j\le n}|\overline{\lambda}_i-\mu_{\overline{\lambda}}||\overline{\lambda}_j-\mu_{\overline{\lambda}}|\right]}$$

$$v)\ \sqrt{\left(\sum_{i=1}^{n}\left|\overline{\zeta}_i-\mu_{\overline{\zeta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\overline{\zeta}_i-\mu_{\overline{\zeta}}\right|\left|\overline{\zeta}_j-\mu_{\overline{\zeta}}\right|+n(n-1)\left[\left|D-\mu_{\underline{\zeta}}\right|\right]^{\frac{2}{n}}}$$
$$\le E\left(D(\underline{I}_{ij})\right)\le\sqrt{2\left[\left(\sum_{i=1}^{n}\left|\overline{\zeta}_i-\mu_{\overline{\zeta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\overline{\zeta}_i-\mu_{\overline{\zeta}}\right|\left|\overline{\zeta}_j-\mu_{\overline{\zeta}}\right|\right]}$$

$$vi)\sqrt{\left(\sum_{i=1}^{n}\left|\overline{\eta}_i-\mu_{\overline{\eta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\overline{\eta}_i-\mu_{\overline{\eta}}\right|\left|\overline{\eta}_j-\mu_{\overline{\eta}}\right|+n(n-1)\left[\left|D-\mu_{\underline{\eta}}\right|\right]^{\frac{2}{n}}}$$

$$\le E\left(D\left(\underline{E}_{ij}\right)\right)\le\sqrt{2\left[\left(\sum_{i=1}^{n}\left|\overline{\eta}_i-\mu_{\overline{\eta}}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\overline{\eta}_i-\mu_{\overline{\eta}}\right|\left|\overline{\eta}_j-\mu_{\overline{\eta}}\right|\right]}$$

4. The Rough Neutrosophic Energy Method

In this section, we present a new approach to multi-criteria decision-making for selecting the best alternative using Rough Neutrosophic matrix energy. Determine the set of k alternatives over m criteria. The alternatives are evaluated by n decision makers. So, we set $A=\{A_1,A_2,\dots A_k\}$, $C=\{C_1,C_2,\dots C_m\}$ and $DM=\{DM_1,DM_2,\dots DM_n\}$

Step 1: The rating values of each alternative on every criterion and the weighted values of m criteria were given by each decision-maker. We take each alternative rating and weight value as a matrix.

Consider the ratings of m criteria given by n decision-makers as a $m\times n$ matrix for weight W.

$$\begin{pmatrix} & DM_1 & DM_2 & \dots & DM_n \\ C_1 & \langle \alpha_{\{11\}},\beta_{\{11\}},\gamma_{\{11\}}\rangle & \langle \alpha_{\{12\}},\beta_{\{12\}},\gamma_{\{12\}}\rangle & \dots & \langle \alpha_{\{1n\}},\beta_{\{1n\}},\gamma_{\{1n\}}\rangle \\ C_2 & \langle \alpha_{\{21\}},\beta_{\{21\}},\gamma_{\{21\}}\rangle & \langle \alpha_{\{22\}},\beta_{\{22\}},\gamma_{\{22\}}\rangle & \dots & \langle \alpha_{\{2n\}},\beta_{\{2n\}},\gamma_{\{2n\}}\rangle \\ \vdots & & & & \vdots \\ C_m & \langle \alpha_{\{m1\}},\beta_{\{m1\}},\gamma_{\{m1\}}\rangle & \langle \alpha_{\{m2\}},\beta_{\{m2\}},\gamma_{\{m2\}}\rangle & \dots & \langle \alpha_{\{mn\}},\beta_{\{mn\}},\gamma_{\{mn\}}\rangle \end{pmatrix} \quad (1)$$

Consider the ratings of n decision-makers over n criteria as a $n\times m$ matrix in alternative A_1.

$$\begin{pmatrix} & C_1 & C_2 & \dots & C_m \\ DM_1 & \langle a_{\{11\}},b_{\{11\}},c_{\{11\}}\rangle & \langle a_{\{12\}},b_{\{12\}},c_{\{12\}}\rangle & \dots & \langle a_{\{1m\}},b_{\{1m\}},c_{\{1m\}}\rangle \\ DM_2 & \langle \alpha_{\{21\}},\beta_{\{21\}},\gamma_{\{21\}}\rangle & \langle \alpha_{\{22\}},\beta_{\{22\}},\gamma_{\{22\}}\rangle & \dots & \langle \alpha_{\{2m\}},\beta_{\{2m\}},\gamma_{\{2m\}}\rangle \\ \vdots & & & & \vdots \\ DM_n & \langle \alpha_{\{n1\}},\beta_{\{n1\}},\gamma_{\{n1\}}\rangle & \langle \alpha_{\{n2\}},\beta_{\{n2\}},\gamma_{\{n2\}}\rangle & \dots & \langle \alpha_{\{nm\}},\beta_{\{nm\}},\gamma_{\{nm\}}\rangle \end{pmatrix} \quad (2)$$

Step 2: Determine the weights of decision makers.
Let $DM_1,DM_2,\dots DM_n$ be the decision makers, they have individual's weights.
Consider $DM_1=\langle x_1,y_1,z_1\rangle$, $DM_2=\langle x_2,y_2,z_2\rangle$, ..., $DM_n=\langle x_n,y_n,z_n\rangle$

Step 3: Determine Rough Neutrosophic Matrix for criteria and alternatives.
The relation between the weight of decision makers and criteria is formed as a Rough neutrosophic matrix for criteria.

$W(C_1DM_1)=$
$\langle \left(min(x_1,\alpha_{11}),max(y_1,\beta_{11}),max(z_1,\gamma_{11})\right),\left(max(x_1,\alpha_{11}),min(y_1,\beta_{11}),min(z_1,\gamma_{11})\right)\rangle$

$$W(C_1 DM_1) = \langle \left(\underline{\alpha}_{11}, \underline{\beta}_{11}, \underline{\gamma}_{11}\right), (\overline{\alpha}_{11}, \overline{\beta}_{11}, \overline{\gamma}_{11}) \rangle \tag{3}$$

$$W = \begin{array}{c} \\ \\ \end{array} \left(\begin{array}{cccc} & DM_1 & \dots & DM_n \\ C_1 & \langle \left(\underline{\alpha}_{11}, \underline{\beta}_{11}, \underline{\gamma}_{11}\right), (\overline{\alpha}_{11}, \overline{\beta}_{11}, \overline{\gamma}_{11}) \rangle & \dots & \langle \left(\underline{\alpha}_{1n}, \underline{\beta}_{1n}, \underline{\gamma}_{1n}\right), (\overline{\alpha}_{1n}, \overline{\beta}_{1n}, \overline{\gamma}_{1n}) \rangle \\ C_2 & \langle \left(\underline{\alpha}_{21}, \underline{\beta}_{21}, \underline{\gamma}_{21}\right), (\overline{\alpha}_{21}, \overline{\beta}_{21}, \overline{\gamma}_{21}) \rangle & \dots & \langle \left(\underline{\alpha}_{2n}, \underline{\beta}_{2n}, \underline{\gamma}_{2n}\right), (\overline{\alpha}_{2n}, \overline{\beta}_{2n}, \overline{\gamma}_{2n}) \rangle \\ \vdots & & & \vdots \\ C_m & \langle \left(\underline{\alpha}_{m1}, \underline{\beta}_{m1}, \underline{\gamma}_{m1}\right), (\overline{\alpha}_{m1}, \overline{\beta}_{m1}, \overline{\gamma}_{m1}) \rangle & \dots & \langle \left(\underline{\alpha}_{mn}, \underline{\beta}_{mn}, \underline{\gamma}_{mn}\right), (\overline{\alpha}_{mn}, \overline{\beta}_{mn}, \overline{\gamma}_{mn}) \rangle \end{array} \right)$$

The relation between the weight of criteria and alternatives is formed as a Rough neutrosophic matrix for alternative.

$$A_1(DM_1 C_1) = \langle \left(min(\alpha_{11}, a_{11}), max(\beta_{11}, b_{11}), max(\gamma_{11}, c_{11})\right),$$
$$\left(max(\alpha_{11}, a_{11}), min(\beta_{11}, b_{11}), min(\gamma_{11}, c_{11})\right) \rangle$$
$$A_1(DM_1\, C_1) = \langle \left(\underline{a}_{11}, \underline{b}_{11}, \underline{c}_{11}\right), \left(\overline{a}_{11}, \overline{b}_{11}, \overline{c}_{11}\right) \rangle \tag{4}$$

$$A_1 = \left(\begin{array}{cccc} & C_1 & \dots & C_m \\ DM_1 & \langle (\underline{a}_{11}, \underline{b}_{11}, \underline{c}_{11}), (\overline{a}_{11}, \overline{b}_{11}, \overline{c}_{11}) \rangle & \dots & \langle (\underline{a}_{1m}, \underline{b}_{1m}, \underline{c}_{1m}), (\overline{a}_{1m}, \overline{b}_{1m}, \overline{c}_{1m}) \rangle \\ DM_2 & \langle (\underline{a}_{21}, \underline{b}_{21}, \underline{c}_{21}), (\overline{a}_{21}, \overline{b}_{21}, \overline{c}_{21}) \rangle & \dots & \langle (\underline{a}_{2m}, \underline{b}_{2m}, \underline{c}_{2m}), (\overline{a}_{2m}, \overline{b}_{2m}, \overline{c}_{2m}) \rangle \\ \vdots & & & \vdots \\ DM_n & \langle (\underline{a}_{n1}, \underline{b}_{n1}, \underline{c}_{n1}), (\overline{a}_{n1}, \overline{b}_{n1}, \overline{c}_{n1}) \rangle & \dots & \langle (\underline{a}_{nm}, \underline{b}_{nm}, \underline{c}_{nm}), (\overline{a}_{nm}, \overline{b}_{nm}, \overline{c}_{nm}) \rangle \end{array} \right)$$

Step 4: In this step, we convert the non-square matrix into a square matrix.

From the above W, the matrix is expressed as 6 matrices which are truth, indeterminacy, false matrix of lower approximation and truth, indeterminacy, false matrix of upper approximation Which are denoted by $W(\underline{T}_{ij})$, $W(\underline{I}_{ij})$, $W(\underline{F}_{ij})$ and $W(\overline{T}_{ij})$, $W(\overline{I}_{ij})$, $W(\overline{F}_{ij})$. Similarly, A_1 matrix expressed as $A_1(\underline{T}_{ij})$, $A_1(\underline{I}_{ij})$, $A_1(\underline{F}_{ij})$ and $A_1(\overline{T}_{ij})$, $A_1(\overline{I}_{ij})$, $A_1(\overline{F}_{ij})$.

$$A_1\left(\underline{T}_{ij}\right)_{n\times m} * W\left(\underline{T}_{ij}\right)_{m\times n} = \begin{pmatrix} \underline{\alpha a}_{11} & \underline{\alpha a}_{21} \cdots & \underline{\alpha a}_{n1} \\ \vdots & \ddots & \vdots \\ \underline{\alpha a}_{1n} & \underline{\alpha a}_{2n} \cdots & \underline{\alpha a}_{nn} \end{pmatrix}_{n\times n} \tag{5}$$

Step 5: Using the definition of Rough Neutrosophic matrix energy, calculate the energy of the matrix. We got six energies for truth, indeterminacy, and false matrices of lower and upper approximation for one alternative.

$$E(A_1) = \langle \left(E\left(A_1(\underline{T})\right), E\left(A_1(\underline{I})\right) E\left(A_1(\underline{F})\right)\right), \left(E\left(A_1(\overline{T})\right), E\left(A_1(\overline{I})\right) E\left(A_1(\overline{F})\right)\right) \rangle \tag{6}$$

Step 6: Continue this process for k alternatives. For each alternative, we got Rough Neutrosophic matrix energies of $E(A_1), E(A_2) \dots E(A_k)$.

Step 7: For ranking the energy values we determine the average values of lower and upper approximation values. Then we get,

$$E(A_1) = \langle E(A_1(T)), E(A_1(I)), E(A_1(F)) \rangle$$
$$E(A_2) = \langle E(A_2(T)), E(A_2(I)), E(A_2(F)) \rangle$$
$$\vdots$$
$$E(A_k) = \langle E(A_k(T)), E(A_k(I)), E(A_k(F)) \rangle$$

Finally, we rank the alternatives according to their truth values. The alternative that has the highest truth energy value will be the best.

5. Numerical Example

We solve the problem by using our proposed method to choose the best place to construct the school building in a particular town. In this problem, the decision makers are Project manager (DM_1), Approval officer (DM_2), Engineer (DM_3), and Public representative (DM_4). The following are the criteria for deciding where to build: C_1- Land Clearance, C_2- Land Title, C_3- Zonal Clearance, C_4- Cost, C_5- Transport Facility, and C_6- Building Plan. The decision-makers choose the best place from the following alternatives based on the above criterion, Place A, Place B, Place C, Place D, Place E, and Place D. The decision makers give their ratings in terms of linguistic variables. IT is shown in Table 1.

Table 1. Linguistic variable for SVNN

S.No	Linguistic Variable	Neutrosophic numbers
1	Very Poor (VP)/ Very low (VL)	$\langle 0.1,0.8,0.9 \rangle$
2	Poor (P)/ Low (L)	$\langle 0.35,0.6,0.7 \rangle$
3	Medium (M)/ Fair (F)	$\langle 0.5, 0.4, 0.45 \rangle$
4	Good(G)/ High (H)	$\langle 0.8, 0.2, 0.15 \rangle$
5	Very Good (VG)/ Very High (VH)	$\langle 0.9, 0.1, 0.1 \rangle$

Step: 1 The decision makers evaluate the criteria and each alternative by the linguistic variable. It is shown in Table 2 and 3 respectively.

Table 2. Weights of Criteria

Criteria	DM_1	DM_2	DM_3	DM_4
C_1	$VG\langle .9,.1,.1 \rangle$	$G\langle .8,.2,.15 \rangle$	$VG\langle .9,.1,.1 \rangle$	$M\langle .5,.4,.45 \rangle$
C_2	$M\langle .5,.4,.45 \rangle$	$G\langle .8,.2,.15 \rangle$	$M\langle .5,.4,.45 \rangle$	$G\langle .8,.2,.15 \rangle$
C_3	$G\langle .8,.2,.15 \rangle$	$VG\langle .9,.1,.1 \rangle$	$G\langle .8,.2,.15 \rangle$	$G\langle .8,.2,.15 \rangle$
C_4	$H\langle .8,.2,.15 \rangle$	$H\langle .8,.2,.15 \rangle$	$F\langle .5,.4,.45 \rangle$	$VH\langle .9,.1,.1 \rangle$
C_5	$M\langle .5,.4,.45 \rangle$	$G\langle .8,.2,.15 \rangle$	$G\langle .8,.2,.15 \rangle$	$M\langle .5,.4,.45 \rangle$
C_6	$VG\langle .9,.1,.1 \rangle$	$VG\langle .9,.1,.1 \rangle$	$G\langle .8,.2,.15 \rangle$	$G\langle .8,.2,.15 \rangle$

Table 3. Ratings in terms of Linguistic variables for each alternative

Alt	DM	C_1	C_2	C_3	C_4	C_5	C_6
A	DM_1	$G\langle .8,.2,.15\rangle$	$VG\langle .9,.1,.1\rangle$	$G\langle .8,.2,.15\rangle$	$F\langle .5,.4,.45\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$
	DM_2	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$VH\langle .9,.1,.1\rangle$	$VG\langle .9,.1,.1\rangle$	$P\langle .35,.6,.7\rangle$
	DM_3	$P\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$H\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$VG\langle .9,.1,.1\rangle$
	DM_4	$M\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$	$H\langle .8,.2,.15\rangle$	$VG\langle .9,.1,.1\rangle$	$G\langle .8,.2,.15\rangle$
B	DM_1	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$P\langle .35,.6,.7\rangle$	$L\langle .35,.6,.7\rangle$	$G\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$
	DM_2	$VP\langle .1,.8,.9\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$L\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$
	DM_3	$P\langle .35,.6,.7\rangle$	$P\langle .35,.6,.7\rangle$	$G\langle .8,.2,.15\rangle$	$F\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$
	DM_4	$G\langle .8,.2,.15\rangle$	$VP\langle .1,.8,.9\rangle$	$G\langle .8,.2,.15\rangle$	$F\langle .5,.4,.45\rangle$	$VP\langle .1,.8,.9\rangle$	$P\langle .35,.6,.7\rangle$
C	DM_1	$VG\langle .9,.1,.1\rangle$	$G\langle .8,.2,.15\rangle$	$VG\langle .9,.1,.1\rangle$	$F\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$G\langle .8,.2,.15\rangle$
	DM_2	$M\langle .5,.4,.45\rangle$	$VG\langle .9,.1,.1\rangle$	$G\langle .8,.2,.15\rangle$	$H\langle .8,.2,.15\rangle$	$VG\langle .9,.1,.1\rangle$	$M\langle .5,.4,.45\rangle$
	DM_3	$G\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$F\langle .5,.4,.45\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$
	DM_4	$VG\langle .9,.1,.1\rangle$	$VG\langle .9,.1,.1\rangle$	$M\langle .5,.4,.45\rangle$	$H\langle .8,.2,.15\rangle$	$G\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$
D	DM_1	$G\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$	$L\langle .35,.6,.7\rangle$	$P\langle .35,.6,.7\rangle$	$VP\langle .1,.8,.9\rangle$
	DM_2	$P\langle .35,.6,.7\rangle$	$VP\langle .1,.8,.9\rangle$	$G\langle .8,.2,.15\rangle$	$F\langle .5,.4,.45\rangle$	$VG\langle .9,.1,.1\rangle$	$P\langle .35,.6,.7\rangle$
	DM_3	$M\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$	$P\langle .35,.6,.7\rangle$	$H\langle .8,.2,.15\rangle$	$P\langle .8,.2,.15\rangle$	$VP\langle .1,.8,.9\rangle$
	DM_4	$P\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$	$VP\langle .1,.8,.9\rangle$	$H\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$M\langle .5,.4,.45\rangle$
E	DM_1	$G\langle .8,.2,.15\rangle$	$P\langle .35,.6,.7\rangle$	$P\langle .35,.6,.7\rangle$	$F\langle .5,.4,.45\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$
	DM_2	$P\langle .35,.6,.7\rangle$	$G\langle .8,.2,.15\rangle$	$VG\langle .9,.1,.1\rangle$	$H\langle .8,.2,.15\rangle$	$M\langle .5,.4,.45\rangle$	$G\langle .8,.2,.15\rangle$
	DM_3	$M\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$	$VP\langle .1,.8,.9\rangle$	$L\langle .35,.6,.7\rangle$	$G\langle .8,.2,.15\rangle$	$P\langle .35,.6,.7\rangle$
	DM_4	$M\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$	$G\langle .8,.2,.15\rangle$	$L\langle .35,.6,.7\rangle$	$M\langle .5,.4,.45\rangle$	$P\langle .35,.6,.7\rangle$

Step: 2 Weights of decision makers.

$DM_1 = G\langle 0.8, 0.2, 0.15\rangle$, $DM_2 = VG\langle 0.9, 0.1, 0.1\rangle$, $DM_3 = M\langle 0.5, 0.4, 0.45\rangle$ and $DM_4 = G\langle 0.8, 0.2, 0.15\rangle$

Step: 3 Determine Rough Neutrosophic Matrix for criteria.

Table 4 shows that the relation between the weight of decision makers and criteria are formed as Rough neutrosophic matrix for criteria

$$W(C_1DM_1) = \langle (\min(0.8, 0.9), \max(0.2, 0.1), \max(0.15, 0.1)),$$
$$(max(0.8, 0.9), min(0.2, 0.1), min(0.15, 0.1))\rangle$$
$$W(C_1DM_1) = \langle (0.8, 0.2, 0.15), (0.9, 0.1, 0.1)\rangle$$

Table 4. Rough Neutrosophic Matrix of Criteria

C	DM_1	DM_2	DM_3	DM_4
C_1	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$
C_2	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.5,.4,.45)\rangle$	$\langle(.8,.2,.15),(.8,.2,.15)\rangle$
C_3	$\langle(.8,.2,.15),(.8,.2,.15)\rangle$	$\langle(.9,.1,.1),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$\langle(.8,.2,.15),(.8,.2,.15)\rangle$
C_4	$\langle(.8,.2,.15),(.8,.2,.15)\rangle$	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.5,.4,.45)\rangle$	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$
C_5	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$
C_6	$\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$\langle(.9,.1,.1),(.9,.1,.1)\rangle$	$\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$\langle(.8,.2,.15),(.8,.2,.15)\rangle$

Table 5 shows that the relation between the weight of criteria and alternatives are formed as Rough neutrosophic matrix for alternative

$$A_1(DM_1C_2) \ = \langle\ \big(min(0.5, 0.9), max(0.4, 0.1), max(0.45, 0.1)\big),$$
$$\big(max(0.5, 0.9), min(0.4, 0.1), min(0.45, 0.1)\big)\rangle$$
$$A_1(DM_1C_2) \ = \langle\ (0.5, 0.4, 0.45), (0.9, 0.1, 0.1)\ \rangle$$

Table 5. Rough neutrosophic matrix of Alternative 1

DM	Rough Neutrosophic values of each criterion for A_1		
DM_1	$C_1\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$C_2\langle(.5,.4,.45),(.9,.1,.1)\rangle$	$C_3\langle(.8,.2,.15),(.8,.2,.15)\rangle$
	$C_4\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$C_5\langle(.5,.4,.45),(.5,.4,.45)\rangle$	$C_6\langle(.8,.2,.15),(.9,.1,.1)\rangle$
DM_2	$C_1\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$C_2\langle(.8,.2,.15),(.8,.2,.15)\rangle$	$C_3\langle(.5,.4,.45),(.9,.1,.1)\rangle$
	$C_4\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$C_5\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$C_6\langle(.35,.6,.7),(.9,.1,.1)\rangle$
DM_3	$C_1\langle(.35,.6,.7),(.9,.1,.1)\rangle$	$C_2\langle(.5,.4,.45),(.5,.4,.45)\rangle$	$C_3\langle(.8,.2,.15),(.8,.2,.15)\rangle$
	$C_4\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$C_5\langle(.5,.4,.45),(.8,.2,.15)\rangle$	$C_6\langle(.8,.2,.15),(.9,.1,.1)\rangle$
DM_4	$C_1\langle(.5,.4,.45),(.5,.4,.45)\rangle$	$C_2\langle(.1,.8,.9),(.8,.2,.15)\rangle$	$C_3\langle(.5,.4,.45),(.8,.2,.15)\rangle$
	$C_4\langle(.8,.2,.15),(.9,.1,.1)\rangle$	$C_5\langle(.5,.4,.45),(.9,.1,.1)\rangle$	$C_6\langle(.8,.2,.15),(.8,.2,.15)\rangle$

Step 4: We convert the non-square matrix into a square matrix.
From table 4 and 5, we expressed the both matrices into 6 matrices. Now we consider the truth lower approximation matrix of both tables

$$A_1\left(\underline{T}_{ij}\right)_{n\times m} = \begin{bmatrix} 0.8 & 0.5 & 0.8 & 0.5 & 0.5 & 0.8 \\ 0.5 & 0.8 & 0.5 & 0.8 & 0.8 & 0.35 \\ 0.35 & 0.5 & 0.8 & 0.5 & 0.5 & 0.8 \\ 0.5 & 0.1 & 0.5 & 0.8 & 0.5 & 0.8 \end{bmatrix}$$

$$W\left(\underline{T}_{ij}\right)_{m\times n} = \begin{bmatrix} 0.8 & 0.5 & 0.8 & 0.8 & 0.5 & 0.8 \\ 0.8 & 0.8 & 0.9 & 0.8 & 0.8 & 0.9 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.8 & 0.8 & 0.8 & 0.5 & 0.8 \end{bmatrix}$$

$$A_1\left(\underline{T}_{ij}\right)_{n\times m} * W\left(\underline{T}_{ij}\right)_{m\times n} = \begin{bmatrix} 2.82 & 3.28 & 1.95 & 2.73 \\ 2.52 & 3.085 & 1.875 & 2.61 \\ 2.46 & 2.92 & 1.725 & 2.505 \\ 2.38 & 2.69 & 1.6 & 2.26 \end{bmatrix}$$

Step 5: Calculating the Energy of Rough Neutrosophic Matrix

Eigen values of the above matrix 9.8836, 0.0176 + 0.0579i, 0.0176 - 0.0579i, -0.0287 and mean of the eigen values is 2.4725.

$$E\left(\underline{T}_{ij}\right) = |9.8836 - 2.4725| + |0.0176 + 0.0579i - 2.4725| + |0.0176 - 0.0579i - 2.4725| + |-0.0287 - 2.4725| = 14.8235$$

Step 6: Similarly, we can find the energy for all the matrices of lower and upper approximation of truth, indeterminacy, and false matrices.

Energy of Rough neutrosophic Matrices

Place A = [(14.8236, 3.6111, 3.8090), (23.7794, 1.1667, 0.9430)]

Place B = [(11.4882, 4.8871, 5.5057), (23.2477, 1.3475, 1.1150)]

Place C = [(16.1570, 2.9861, 3.0163), (24.2484, 1.1979, 1.0370)]

Place D = [(11.0676, 4.9059, 5.4638), (22.5909, 1.3974, 1.1322)]

Place E = [(13.4987, 4.3102, 4.6900), (22.1196, 1.4901, 1.2752)]

Step 7: Calculate the average value of the lower and upper approximation of energy of Rough Neutrosophic sets, then the ranking of alternatives will be decided by the truth values.

Average energy of each Alternative is given below

Place A = [(19.3015, 2.3889, 2.376)]

Place B = [(17.3679, 3.1173, 3.3103)]

Place C = [(20.2027, 2.092, 2.0266)]

Place D = [(16.8292, 3.1516, 3.298)]

Place E = [(17.8091, 2.9001, 2.9826)]

The Average energy of truth and ranking order of alternatives presented in Table 6.

Table 6. Ranking order

Alternatives	Truth Energy	Ranking Order
Place A	19.3015	II
Place B	17.3679	IV
Place C	20.2027	I
Place D	16.8292	V
Place E	17.8091	III

The ranking order of the alternatives is C > A > E > B > D. Place C is the best location to build the school construction in the town.

6. Conclusion

The energy of the matrix helps to determine the matrix's weight. We apply this idea of energy to the Rough neutrosophic matrix. The Energy of Rough Neutrosophic Matrix contains truth indeterminacy and false energy for the lower and upper approximations of each matrix. The final energy was determined by averaging the lowest and upper approximations of each energy. In that, the ranking of alternatives is evaluated using truth value. In our taken problem, the decision-maker chooses the perfect spot for the construction of the school. The building should be constructed at Place C. It satisfies all requirements. As a result, the Energy of Rough Neutrosophic Matrix will be used in every situation and our proposed energy method helps to solve the multi-criteria decision-making problems. Compared to other MCDM methods our presented method simplifies the work and also give more effective result. Further, we will extend the Rough neutrosophic energy concept to other types of Rough neutrosophic matrices such as interval-valued, multi-valued and so on.

Author Contributions: Conceptualization, J.S.M. and D.G.; methodology, J.S.M. and D.G.; software, J.S.M. and D.G.; validation, J.S.M. and D.G.; formal analysis, J.S.M. and D.G.; investigation, D.G.; writing—original draft preparation, J.S.M; writing—review and editing, D.G.; visualization, J.S.M. and D.G.; supervision, D.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Acknowledgments: We are grateful to Vellore Institute of Technology, Vellore for giving us this opportunity.

Data Availability Statement: Not Applicable.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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