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DETECTING BUSINESS CYCLES FOR HUNGARIAN LEADING AND COINCIDENT INDICATORS WITH A MARKOV SWITCHING DYNAMIC MODEL TO IMPROVE SUSTAINABILITY IN ECONOMIC GROWTH

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Abstract: *This paper applies the hidden Markov switching dynamic regression (MSDR) model to estimate transition probabilities of the Hungarian GDP between recessionary and expansionary periods. The transition probabilities are then compared to the OECD Hungarian binary business cycle indicator to assess the predictive power of the model. The paper proposes a linear model with a mean and a homoscedastic component. The level of symmetricity between the GDP and business cycles is explained by the panel data variables (Unemployment rate, IPI index, Inflation, BUX year-on-year change, and 10-3 Year sovereign bond yield spreads). It is assumed in this paper that by extending the model to encompass an exogenous variable listed in the panel data, essentially making the model bivariate, the maximum likelihood function would capture the business cycle more accurately. The results show that by plugging the unemployment rate as the exogenous variable in the regression, our model's accuracy is 70%.*

Keywords: *Markov switching dynamic regression model, business cycles, recession.*

1. Introduction

In their seminal work, (Burns & Mitchell, 1946) have framed the features of a nation's aggregate economic activity into the theoretical concept of a business cycle

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consisting of expansions and contractions. According to (Shaw, 1947)this allowed for the first time in the history of econometrics to decompose economic time series into cyclical, seasonal, trend and error components, opposing the pre-existing Keynesian unemployment equilibrium theory and its doctrine. While the idea of business cycles has been subject to rigorous analysis and criticism by Koopmans' (1947), and later, by Stock's (1987) revision that statistically disproved the existence of a macroeconomic time scale, it nonetheless introduced a new field of research in econometrics – business cycle forecasting.

Empirical studies concerned with determining relationships between economic variables and business cycles branched into: dynamic factor modelling approach aimed at capturing comovements of coincident economic indicators (CEI), leading economic indicators (LEI) and finding composite leading indicators (CLI) for a nation's economy (Stock et al., 1991); Hamilton (1989) extension on Goldfeld and Quandt (1973) Markov switching model; and, more recently, a synthesis of the aforementioned regime switching and dynamic factor models by Diebold and Rudebusch (1996) and Chauvet (1998).

Changes in the macroeconomic environment in the form of rapid declines in output, hyperinflations and economic crises have been challenging to incorporate into a stationary linear model, due to the presence of structural breaks. When expanding the observed time horizon, it becomes evident that periods of growth and decline in the economy are recuring and cyclical. Therefore, the model in question has to incorporate a parameter that would take on different values depending on the discrete number of 'regimes' the system can theoretically be in. This enables to capture the structural breaks inherent to the system (Piger, 2009). In Hamilton's regime switching hidden Markov model, the time series is controlled by a parameter vector that changes depending on an unobserved state variable that follows the evolution of a first-order Markov chain. The inherent property of a Markov chain is that the future value depends on its immediate value alone and not its previous values – this allows to perform accurate short-term out-of-sample forecasting and identifying 'turning points' of business cycles. There has been ample evidence on the successful implementation of univariate Markov regime-switching dynamic factor models for characterization of business cycle dynamics, particularly with datasets of developed economies. From the seminal work of Hamilton, which was based on Goldfeld & Quandt's (1973) and Neftçi's (1984) analysis of unemployment and business cycle asymmetry to (Filardo, 1994) time-varying transition probabilities, augmented with (Layton & Smith, 2000) signalling system – the Markov regime switching model and its extensions has generated many promising results for time series of developed countries. As of writing this research, there have been few studies of business cycle modelling for Eastern European countries and developing economies. Industrial business cycles were determined through a Markov-switch method for Romania, Poland and the Czech Republic by (Spulbăr et al., 2012). Bandholz (2005) applied the univariate Markov regime-switching model to the industrial production index of Poland and Hungary as well as the BUX composite stock index, while, more recently, Siničáková (2017) analyzed the level of business cycle synchronization of the Visegrad group countries with the Euro area through a Markov-switching autoregressive model. Leon Li et al. (2005) found that the regime switching models failed to characterize the business cycle of newly industrialized South Korea and Taiwan. What is common in most of the research is that a univariate Markov switching model is applied to the economic time series.

The development of a unified recession likelihood indicator has long been the aim of many econometricians and policy makers. Stock and bond markets of emerging economies, while less efficient than their developed counterparts, have been the primary subjects for testing the Markov switch models, since they offer an insight into forward earning expectations and the rigidity of credit markets. Markov regime switching models, and probit models, however, are not the only tools used in corporate decision making and economic forecasting. For example, in Baydaş and Elma (2021), multi-criteria decision-making method framework is being applied to determine the correlation between a company's share price and increase in earnings. Further models include rough or, sometimes referred to as 'fuzzie' set modelling. In the research by Sharma et al. (2020) through the total roughness measurement statistic, the authors determine which among the Holt-Winters, Grey and SARIMA (Seasonal Auto-regressive moving average) models are the most accurate for forecasting univariate data sets. For multivariate data sets Sagar et al. (2021) raise the problem of the cost of big data gathering and maintenance, and how an additional independent variable in the multivariate data set can only be of an incremental advantage in estimating the dependent variable. Regression analysis is still an incredibly popular technique, and it is one chose by the authors complemented with an original MIPA algorithm that outputs the lowest RMSE (Root mean square error) and residuals.

However, as it is pointed out by Blanchard and Quah (1989) and later reaffirmed in Kuan (2002) work, a composite economic variable, such as the industrial production index, the GDP or the GNP is affected by multiple disturbances. Therefore, modelling the time series through a univariate autoregressive Markov regime switching process could yield inaccurate results. This research adopts a bivariate regression model to determine links between the Hungarian GDP growth rate and a set of key macro leading, coincident and lagging indicators, such as the unemployment rate, inflation, industrial production index, BUX composite stock market quarterly returns and 10-3-year yield spreads of Hungarian sovereign bonds. The estimation of the hidden Markov model will allow to explain the changes in the GDP using the changes in the individual macro indicators. The model is built in Python IDE and in addition to the state probability matrix and the visualization of the probabilities of regimes, a state probability distribution is plotted and analyzed in Section 5 of the research.

The objective of this paper is to assess the usefulness of common recession macroeconomic indicators in predicting changes in the GDP for the Hungarian economy through implementing a hidden Markov switching dynamic regression model, extending the initial approach of Hamilton. Thereby, the contribution of this work is threefold: first, it adds to the empirical research on developing countries, since the object of the study is the Hungarian economy; secondly, an exogenous variable is included in the dynamic factor regression model to better explain the shocks of the composite GDP indicator and thirdly, a program listing is included for future improvement and reference.

The rest of the research is structured as follows: section 2 presents a brief literature review of the application of Markov chains in econometrics, the methodology section dissects the Markov regime-switching model specifications. Section 4 provides a description and the reasoning for the choice of the macro indicators. Section 5 presents the results obtained from the model and interprets them. In section 6 the results are critically reviewed and explained in detail. Section 7 draws conclusions and possibilities of future research.

2. Literature review

Before the introduction of Markov regime-switching models in econometrics, the characterization of the relationship between economic activity indicators, such as the GNP or the GDP with the business cycle, have chiefly been through the implementation of autoregressive integrated moving average (ARIMA) models. Simultaneously, a cointegration approach of Granger (1969) has been developed as a means of determining if the lagged values of a time series are useful in predicting the future values of the reference series. As outlined by Hamilton, previous works do not consider the fact that the time series don't follow a linear stationary process, and therefore, the approaches are unsuitable for modelling regime changes in the time series – inherent to business cycles.

In their critical work, Goldfeld and Quandt (1973) employ a Markov switching regression to determine the transition matrix of variable (z) and derive the maximized likelihood function $r(z)$ to characterize the persistence tendencies of supply-demand and the probability of regime prevalence and regime changing. Neftçi's algorithm, is adopted and formalized by the benchmark model of Hamilton, where the economy is between two regimes governed by a two-state first-order Markov algorithm - the methodology which is applied is a Markov-switching autoregression model. The regime switch is dependent only on the previous state before switching occurs, this means that the model does not require any prior information from the system. The turning point, where the regime inflection occurs, can therefore be determined. Further specifications of the model are found in the succeeding part of the research.

Modern applications of Markov dynamic factor models include three-state models for enhanced sensitivity as in Carstensen et al. (2020) for German business cycles. Some works apply the quadratic probability score to estimate the accuracy of the model. Traditionally, the underlying models for the Markov switches range from AR (1) or AR(2) models as in McGrane Michael (2022) to Naïve Bayes model as in Davig and Hall (2019).

Contemporary studies of Markov models applied to emerging market indicators for recession prediction include the work of Afreen (2021), where an AR (1) model of Bangladeshi financial indicators is devised, as well as the works of Tuaneh et al. (2021), where the transition probabilities of the Nigerian economy are examined from the perspective of export and import. The authors of the latter apply the classic Markov Switch VAR model, first applied in Hamilton's seminal work, which in principle is like AR models.

At the same time Markov models aren't only applied to measure the possibility of the economy going into a recession. It is also possible to estimate the likelihood of an economy going into a recession based on another economy's performance. In the EUs integrated economy, if one country defaults on debt or goes into a recession, it may very well drag the rest of the countries in one. Poon and Zhu (2022) studies UK and US concurrent recession probabilities and finds that as the number of countries increases, the model becomes more accurate.

The most prominent literature on Markov regime switching models is usually found for developed countries. As of the publication of this paper, empirical literature on the application of Markov regime-switching dynamic factor models for Central and Eastern European countries is limited to the study of asynchronous behavior of individual business cycles with the Euro area, primarily inspired by a common aim of discrediting a uniform monetary policy and further expansion of the Euro area. While Bandholz (2005), Di Giorgio (2016), Siničáková (2017), applied the Markov switching model for panel data, Artis et al. (2004) and Darvas Zsolt and Szapáry György (2008) used the Hodrick-Prescott filter to measure synchronization. For both methodologies the hypotheses of CEEC and Euro Area business cycle independence have been rejected. A useful extension noted by Di Giorgio (2016) would be considering other variables within the Markov switching vector autoregression model. For stock markets in the CEEC, Moore and Wang (2007), Linne (2002) and Krozlig (1997) works feature the application of Markov switching for weekly stock index returns. The authors use high and low volatility states for regime switching. Recently, a Markov switches have been applied for the Turkish economy, in works of Balcilar et al. (2015) and Bilgili et al. (2020), where the relationships between globalization and environmental sustainability have been examined. In Hoque and Zaidi (2019), for instance the implementation of economic policy on stock market returns had been analyzed. However, it is determined that the level of correlation of an individual countries' endogenous variable with the greater Euro Area business cycle is not uniform across the set of panel data presented in literature. For instance, Moore and Wang determine that in case of Hungary, before the European Union integration, a low volatility regime persisted in the stock market, while in Poland, Slovenia, Slovakia and the Czech Republic, inflection points indicating a regime shift have been observed. This breaks the uniformity in business cycle characterization for other member states. Considering the various qualitative results obtained from the analysis of the literature, this research estimates the model only for Hungary.

Further evidence of the application of Markov switching dynamic factor models to the time series of emerging economies is found in Petreski (2011) work, where the inflation targeting policy effectiveness is analyzed through the scope of exchange rate pegging in Hungary and the Czech Republic. In the estimation equation the authors consider both endogenous and exogenous variables that enter the system. In Kuan's and Balanchard's works a bivariate regression is specified following the reasoning that supply and demand shocks on markets can't be characterized by GDP or GNP regime shifts alone. For Taiwan Chen and Chen (2000) estimates a much stronger identification capability of multivariate Markov-switching models for GDP, consumption expenditure, gross capital formation and real exports

3. Methodology

Before the research moves forward with the specifications of the hidden Markov regime-switching model, it is imperative to understand the underlying basics of this statistical tool – the Markov chain and furthermore, to understand why this model is governed by a process that has no memory. Detailed introduction of Markov Switching models can be found in Hamilton (Kim Chang-Jin & Nelson Charles R., 2017). A comprehensive algorithm set up guide in MATLAB is presented by Perlin Marcelo (2015). The notation as well as the equations are adopted from Hamilton (1989), Perlin (2015) and Date (2022).

Let's illustrate how a Hidden Markov Model can be used to represent a real-world data set. Let we have some real-world economic data represented in Figure 1.

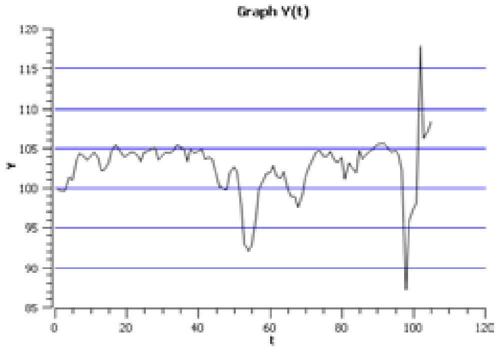

Figure 1. this graph represents data of Hungarian GDP corresponding to the same period of previous year in % - y(t), instead of dates in t I used just an arbitrary serial number. Source: compiled by author

The above graph shows large scale regions of positive and negative growth together with abrupt deviations – crises. This positive and negative growth could be, for example, expansion and contraction of the economy, the ups and downs of the business cycle.

It is hypothesized that: (i) the unknown stochastic process changes the behavior of the indicator, so that the specific characteristics of the behavior are classified into regimes; (ii) there are two regimes in the Hungarian economy denoted as 1 and 2, which represent an expansion and a recession correspondingly; (iii) Every indicator can be represented in two-regime model through the maximum likelihood estimation function; (iv) the maximum likelihood function is more accurate if exogenous variables are added into the model; (v) the regime shifts of the Hungarian GDP in a multivariate model correspond to the business cycles determined by the OECD.

Finding the turning point in future effectively means we could find the starting point of the recession on the GDP time series. While modelling the panel data, we will consider a regression model that is a mixture of the following two random variables: the observable random variable y(t), which would be used to represent the observable pattern (the observable economic variable itself), and a hidden random variable m(t) which is assumed to change its state or regime, and each time the regime changes, it affects the observed pattern of *y(t)*. In other words, a change in value of *m(t)* impacts the mean and variance of *y(t)*. This is the primary idea behind Hidden Markov Models. For convenience of the reader, we present here the short summary of the relevant theory.

Let's assume that *m(t)* switches between two regimes 1 and 2. It means that we start to use the simplest Markov chain for variable *m(t)* – the two state Markov chain. It is obvious *m(t)* is a 'hidden' random variable – it is 'hidden' since we do not know what state the system is in (what regime is in effect), and it is random since we do not know when it would be changed. The two states simple Markov chain is simply a minor generalization of the scheme of independent trials or the so-called Bernoulli scheme.

As it is well-known, the state transition probability matrix P in this case has the form:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad (1)$$

where p_{11} – is the probability of transition from state *1* to the same state *1.* In other words, p_{11} probability equals to the conditional probability that event $m(t+1)=1$ will occur at $t+1$ moment if at the moment t event $m(t)=1$ has occurred, and this probability does not depend on the events that occurred at earlier moments in time: $p_{11}=P(m(t+1)=1|m(t)=1)$. Likewise for other probabilities: $p_{12}=P(m(t+1)=1|m(t)=2)$, $p_{21}=P(m(t+1)=2|m(t)=1)$, $p_{22}=P(m(t+1)=2|m(t)=2)$. Since the Markov process needs to be in some state at each time step, it follows that: $p_{11} + p_{12} = 1$, and $p_{21}+p_{22}=1$. Therefore, we can rewrite the probability matrix as follows:

$$P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}, \quad (2)$$

So, for the definition of our scheme we need only two values: p_{11} and p_{22}. Now for our two state Markov chain random variable $m(t)$, which gains values from set {1,2} we can write down the probability distribution function at time t, that is the unconditional probability of the system being at a certain state. It must be a two-component vector $D(t)$ with components: $d_1(t)$ – the unconditional probability that at the moment of time t the system is in the state *1*, and $d_2(t)$ – the unconditional probability that at the moment of time t the system is in the state *2.*

$$D(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} P(m(t)=1) \\ P(m(t)=2) \end{bmatrix}. \quad (3)$$

A Well-known result is that if we start with some prior (initial) probability distribution for m(0), D(0) then D(t) can be computed by simply matrix multiplying P with itself t number of times and multiplying D(0) by the matrix product P^t:

$$D(t) = D(0) \cdot P^t. \quad (4)$$

So far, we have the useful formulae of two-state Markov chain model, although one must admit, we do not know the exact time steps at which $m(t)$ makes the transition from one state to another, and we also do not know the transition probabilities P for the model.

Now we describe shortly the Markov Switching Dynamic Regression used in the current investigation. Let $y(t)$ be an observable time dependent economic indicator being explained and $x_i(t)$, $i=1,2,\ldots n$, is some observable economic indicator which explains $y(t)$. We restrict ourselves here with general linear regression model:

$$Y = X\theta + \varepsilon, \quad (5)$$

where $Y = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$ vector of sequence of y(t) values at different time moments such that $y_1=y(t_1),\ldots,y_p=y(t_p)$, $X = \begin{bmatrix} 1\ x_{11} \ldots x_{1n} \\ 1\ x_{21} \ldots x_{2n} \\ \ldots\ \ldots\ \ldots\ \ldots \\ 1\ x_{p1} \ldots x_{pn} \end{bmatrix}$ is matrix containing the explanatory variables data with the first column filled with ones, for x_{ij} value index i means the number of the point, while index j means the number of the explanatory variable

($x_{11}=x_1(t_1)$, $x_{21}=x_1(t_2)$,…,$x_{pn}=x_n(t_p)$)), $\theta = \begin{bmatrix}\theta_0\\ \vdots \\ \theta_n\end{bmatrix}$ the vector θ contains the coefficients of the linear model. The aim of the regression model is to determine θ. As usual the $\varepsilon = \begin{bmatrix}\varepsilon_0\\ \vdots \\ \varepsilon_p\end{bmatrix}$ is the vector of residual error which supposed to be a normally distributed random variable with zero mean:$\varepsilon \sim N(0,\sigma^2)$. We assume here the homoskedasticity of ε.

Now let we see how one can mix the simple regression model with two state Markov process. Depending what Markov state is in effect the linear regression coefficients set θ_i which describes data depends on what Markov state is in effect at the moment t. Namely:

$$\begin{cases} Y = X\theta^1 + \varepsilon_1, when\ m = 1, \\ Y = X\theta^2 + \varepsilon_2, when\ m = 2. \end{cases} \tag{6}$$

Here Y – vector of y explained data set, X – $n{\times}p$ matrix of explication data set (n –variables at p time moments), θ^m – vector of linear regression coefficients which corresponds to regime m (here m –aC is the index and not a power), and random variables $\varepsilon_m \sim N(0,\sigma_m^2)$. Now we calculate for time moment t_i

$$[1\ x_{i1} \dots x_{in}\ ] \cdot \begin{bmatrix}\theta_0^1 & \theta_0^2\\ \vdots & \vdots \\ \theta_n^1 & \theta_n^2\end{bmatrix} = [y_i^1\ y_i^2] \tag{7}$$

$$\overline{y_i} = y_i^1 \cdot P(m = 1) + y_i^2 \cdot P(m = 2) \tag{8}$$

or

$$\overline{y_i} = [y_i^1\ y_i^2] \cdot D(t_i) \tag{9}$$

Where,

$$D(t_i) = \begin{bmatrix}P(m(t_i) = 1)\\ P(m(t_i) = 2)\end{bmatrix} \tag{10}$$

Now we have to estimate somehow the $\hat{\theta} = \begin{bmatrix}\theta_0^1 & \theta_0^2\\ \vdots & \vdots \\ \theta_n^1 & \theta_n^2\end{bmatrix}$ matrix of regression coefficients and the probability matrix for Markov chain P, and eventually the variance σ^2 of the dependent variable y. We will use the Maximum Likelihood Estimation (MLE) method (for proper estimation of the results we will compare it with the Expectation Maximization method). MLE, which finds the values of P, $\hat{\theta}$ and σ^2, would maximize the joint probability density of observing the entire data set y. In other words, we have to maximize the following product:

$$L = \prod_{i=1}^{p} f\big(y(t_i)\big) \tag{11}$$

Since $\varepsilon_m \sim N(0,\sigma_m^2)$ it is obvious and convenient to assume that

$$f\big(y(t_i)\big) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{y_i - \overline{y_i}}{\sigma}\right)^2} \tag{12}$$

So, we obtain formulae for calculations:

$$L\left(\sigma,\theta_l^k,P(m=1),P(m=2)\right)=\prod_{i=1}^{p}\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\left(\frac{y_i-\overline{y_i}}{\sigma}\right)^2} \tag{13}$$

$$\overline{y_i}=y_i^1\cdot P(m=1)+y_i^2\cdot P(m=2), \tag{14}$$

$$y_i^1=[1\ x_{i1}\ \dots x_{in}\ ]\cdot\begin{bmatrix}\theta_0{}^1\\ \vdots\\ \theta_n{}^1\end{bmatrix}, y_i^2=[1\ x_{i1}\ \dots x_{in}\ ]\cdot\begin{bmatrix}\theta_0{}^2\\ \vdots\\ \theta_n{}^2\end{bmatrix}. \tag{15}$$

Providing the standard maximization procedure and solving the following system of equations we obtain the estimated parameters we were looking for:

$$\begin{cases}\frac{\partial L}{\partial\sigma}=0\\ \frac{\partial L}{\partial\theta_l^1}=0, l=1,\dots,n\\ \frac{\partial L}{\partial\theta_l^2}=0, l=1,\dots,n\\ \frac{\partial L}{\partial P(m=1)}=0\\ \frac{\partial L}{\partial P(m=2)}=0\end{cases}. \tag{16}$$

To obtain numerical results from observed data we created a python program, since it has a huge functional library, although there are a lot of other options including highly developed mathematical programs like Matlab™ or Mathematica™.

3.1 Implementation in Python IDE

In Python IDE, to plot the Markov switch probabilities π_t we first import the following libraries: pandas, numpy, pyplot and statsmodels.

Then we can load the datasets, which, in case of this research, are stored on the github repository. It is advisable to first plot the datapoints and visualize the time series to visually confirm possible turning points and regime switches.

The y_t dependent variable, further regarded in the research as the endogenous variable, is represented by the $GDP\ \%Chng$ variable. The regression variable – the one that we observe, is given by the individual panel data time series.

Following the statsmodels manual developed by Perktold et al. (2022), to build perform the switching regression, we build and train the two-state Markov switching dynamic regression model by first assigning a variable to it:

Msdr_model = sm.tsa.MarkovRegression(endog=df['$GDP\ \%Chng$'], k_regimes=2, trend='c', exog=df['X'], switching_trend=True, switching_variance=True)

The "sm.tsa.MarkovRegression" takes in the endogenous variable, number of specified regimes, adds an intercept – trend 'c', the exogenous variable, the boolean of the switching trend and exogenous variable and outputs a table of summaries that can be viewed by running the command: "Msdr_model_results = msdr_model.fit()" and " Print(msdr_model_results.summary())."

4. Data and selection of economic variables

To satisfy the objective of the research of estimating the business cycle, the research employs. The data were collected from the OECD quarterly national accounts, the IMF International Financial statistics database, the central statistical office of Hungary (KSH) as well as the Budapest stock exchange (BÉT) data warehouse of. The binary Hungarian recession indicator is based on the OECD recession indicator retrieved from the Federal Reserve Bank of St. Louis database. 1 is assigned to recessionary periods and 0 is assigned for recovery/expansionary periods. (Arturo Estrella & Frederic Mishkin, 1998)propose a set of 6 indicators of economic activity, however, due to low complexity of Hungarian financial markets, this research instead focuses on classical indicators such as the unemployment rate, industrial production index, the inflation rate and the BUX composite stock market index returns. Only one of the proposed indicators is considered in this research – the 10 year and 3 year government bond yield spreads. Table 1 provides some descriptive statistics of the selected variables.

Table 1. Descriptive statistics of selected Hungarian economic time series. Source: Own calculation

	GDP %	Unemployment rate	Industrial production index	Inflation rate	BUX	10Y-3M yield
N	105	105	105	105	105	93
μ	2.60	7.02	5.10	6.21	20575	0.10
σ^2	3.58	2.33	7.71	5.50	12523	1.42
min	-12.8	3.3	-22.9	-1	2635.0	-3.6
max	17.8	11.1	21.1	27.9	54197	2.3

The data sample covers the period between Q1-1996 to Q1-2022, with a quarterly frequency totaling 105 observations. The 10-year and 3-year yield time series samples cover a slightly smaller timespan of Q1-1999 to Q1-2022. The seasonally adjusted time series are represented in a quarterly frequency. The choice of the unemployment rate as a lagging indicator in the scope of a Markovian model is inspired by its analysis in many empirical works on the US economy. In a recent study, McGrane proved that there's an asymmetry between business cycles and unemployment, whereby the unemployment rate rises faster in recessions than it falls during expansions, supporting the New Keynesian model. Nevertheless, for sake of obtaining the regime-switching probabilities of the unemployment rate, this research follows the standard notion of the business cycle and unemployment correlation and relaxes assumptions of wage stickiness. The choice for the industrial production index as an indicator of business cycle dynamics is supported in Artis et al. (2004) findings of a common cycle independent of the industrial sector, and by Medhioub Imed (2015) three-state regime model, as an example of a developing economy. Inflation signifies a change in the price of consumer goods – this happens because of an increase in money supply, due to monetary policy enacted to combat exogenous or endogenous shocks and to keep credit available in the economy. High inflation rates signal greater probabilities of recessions, hence a direct link to business cycles. The stock market is by far the most popular leading indicator. Since the current price of a stock doesn't only reflect future earnings expectations but also consumer sentiment, it is highly correlated with business cycles. The BUX composite share price index is used in

Bandholz's univariate model. However, thanks to a wider timeframe and greater availability of observations in this research, the statistical inferences that can be drawn from the model become more reliable. The difference between the 10-year and 3-year Hungarian sovereign bond yields indicates the investors' outlook on future economic conditions. The wider the spread, the steeper the yield curve is, and the more positive the outlook on the economy. Conversely, the tighter the spread, the less confident investors are in the economy. Negative 10-year and 2-year yield spreads are notorious for predicting every single recession in the US. As a result, the research. The reference series the cyclical dynamics of which the research aims to predict is the quarterly GDP growth rate.

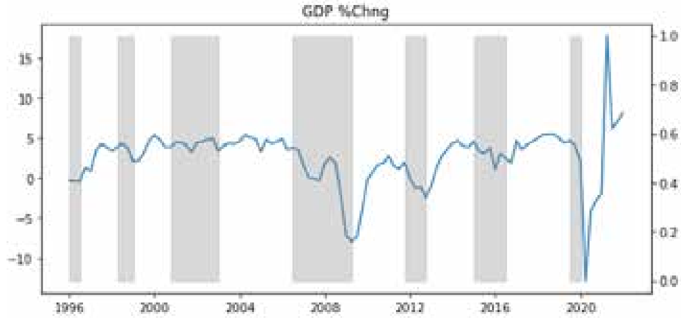

Figure 2. GDP quarterly growth rate superimposed on OECD recession periods. Source: Own calculations (OECD)

To better understand the reference series, it is appropriate to provide a brief description of the recession periods graphed in Figure 2. The grey areas represent peak-to-through recession periods in the Hungarian economy following the OECD recession indicator. Overall, seven recessionary periods are identified in the dataset. As a result of the fiscal stabilization policy in 1995 the GDP growth rate decelerated, causing a mild recession. The 1998 recession was caused by a sharp decline in demand because of the Russian financial crisis. Following the burst of the dot com bubble in 2001, foreign demand decreased yet again causing a recession. The inflation rate of the Forint sharply dropped during this period. The 2008 global financial crisis caused by subprime lending, risky financial products, and the housing market collapse in the US resulted in a significant drop in the Hungarian GDP. Being on the brink of default, Hungary was bailed out by the IMF and the European Union. In 2012 the eurozone sank into a debt crisis that resulted in an unfavorable economic environment and low domestic demand. The mild decline in 2016 was partly due to a reduction in European Union financing and partly due to a weaker automobile industry. In 2020 the economy witnessed the sharpest decline in GDP growth in its history because of nationwide lockdowns caused by the COVID-19 pandemic. Altogether, the selected indicators form a strong base for testing the methodology for the Hungarian economy. The data can be retrieved in the following repository: "https://github.com/albertmolnar/Markov-chain"

5. Empirical results for Hungarian economic variables

Regime 1 and 2 models of the GDP as y(t) and the Unemployment rate m(t):

Regime 1: GDP %Change = $6.24 - 0.33 Unemployment\ rate + 0.58$;

Regime 2: GDP %Change = $5.33 - 0.67 Unemployment\ rate + 20.49$

Table 2 shows the results of the test. The transition probability matrix is defined as: $P = \begin{bmatrix} 0.9267 & 0.0733 \\ 0.0983 & 0.9017 \end{bmatrix}$, where the expected duration for regime 1 is 13.63 quarters, while the expected duration of regime 2 is 10.17 quarters, which are visually represented in Figure 3. For additional reference on the Markov switching model result, Appendix A-E provides the state space probability distributions.

Table 2. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the unemployment rate. Source: Own calculations based on FRED and OECD data

U	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	6.24	-0.33	0.58	0.00	0.00	0.00	0.93	0.09
Reg.2	5.33	-0.67	20.49	0.017	0.019	0.00		

Given the 50% chance of regimes 1 and 50% chance of regime 2, after 100 iterations, the state probability of switching to state 2 from state one reaches 0.427156. the calculated probability of remaining in a low state while being in one is 0.57284.

The smoothed state probabilities D(t) at time t, where t = 1 quarter, and the state is equal to 2 (a recession) are plotted below. Figure 3 illustrates a side by side comparison of the smoothed probability of recessions compared to actual recessions and The GDP % change.

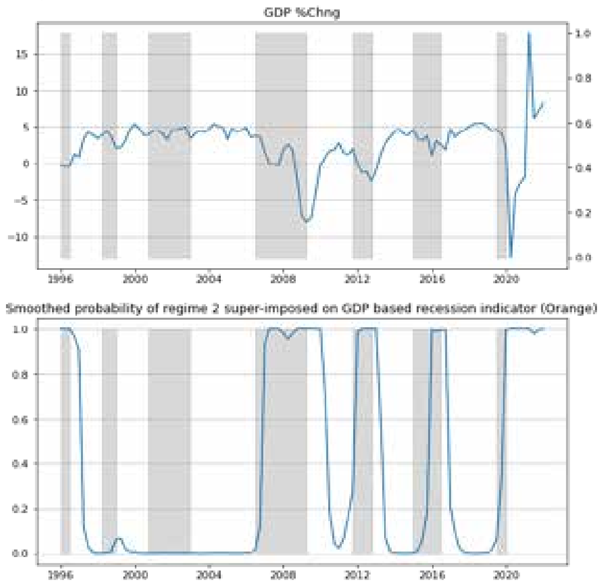

Figure 3. Smoothed probabilities of being in a recession superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

We lag the explaining variable m(t) by one quarter to see if its past values could be more efficient in producing the probability estimate.

Regime 1 and 2 models of the GDP as y(t) and the Lagged unemployment rate m(t) in Table 3are as follows:

Regime 1: GDP %Change = $5.44 - 0.18\text{Lagged}Unemployment\ rate + 0.34$;

Regime 2: GDP %Change = $3.56 - 0.37\text{Lagged}Unemployment\ rate + 19.35$

From Table 3 the transition probability matrix is: $P = \begin{bmatrix} 0.9206 & 0.0794 \\ 0.9214 & 0.0786 \end{bmatrix}$.

Table 3. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the 1 quarter lagged unemployment rate time series. Source: Own calculations based on FRED and OECD data

Lag'd U	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	5.44	-0.18	0.34	0.00	0.00	0.00	0.9	0.07
Reg.2	3.56	-0.37	19.4	0.089	0.013	0.00		

Using the "msdr_model_results.expected_durations" function, we determine that the expected duration of regime 1 is 12.60 quarters, while the expected duration of regime 2 is 12.73 quarters.

The probability of a recession given by the hidden regime switching process of the lagged y(t) is specified in Figure 4.

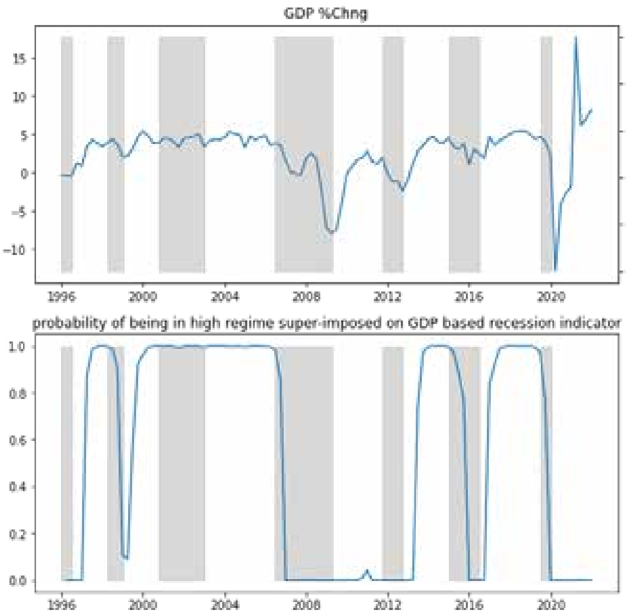

Figure 4 Smoothed probabilities of a recession, with lagged unemployment variable as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regression models for the regimes are as follows:

Regime 1: GDP %Change = $4.41 - 0.017Inflation\ rate + 0.407$;

Regime 2: GDP %Change = $0.40 + 0.031Inflation\ rate + 19.60$

From Table 4, the probability matrix is defined as: $P = \begin{bmatrix} 0.9206 & 0.0794 \\ 0.0748 & 0.9252 \end{bmatrix}$, the expected duration for regime 1 is 12.60 quarters, the expected duration of regime 2 is 13.36 quarters visualized in Figure 5.

Table 4. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the inflation rate time series. Source: Own calculations based on FRED and OECD data

Inflation	θ_0	θ_1	σ^2	P>|z| θ_0	P>|z| θ_1	P>|z| σ^2	p_{11}	p_{21}
Regime 1	4.41	-0.0	0.40	0.00	0.35	0.00	0.92	0.07
Regime 2	0.40	0.03	19.6	0.67	0.767	0.00		

The initial D(t) is set to 0.5 and 0.5 for both regimes. Following the same 100 iterations steady state for the 1-2 regime switch is attained at 0.48508, the probability remaining in state 1 is 0.514915.

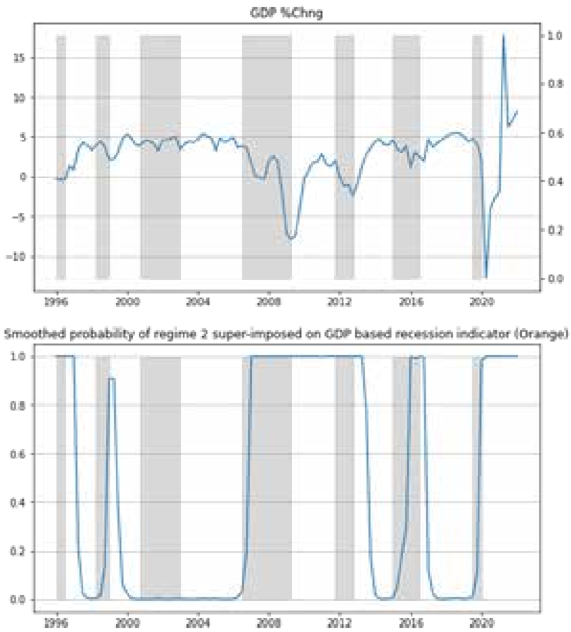

Figure 5. Smoothed probabilities of a recession, with inflation rate as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

From Table 5, the regime specific equations are:
Regime 1: GDP %Change $= 4.42 - 0.018 \text{Lagged} Inflation\ rate + 0.407$;
Regime 2: GDP %Change $= 0.6702 - 0.0047 \text{Lagged} Inflation\ rate + 20.03$

Probability matrix is defined as: $P = \begin{bmatrix} 0.9207 & 0.0793 \\ 0.0761 & 0.9252 \end{bmatrix}$, the expected duration of regime 1 is 12.60 quarters, while the expected duration of regime 2 is 13.36 quarters – Figure 6 illustrates the results. Lagged inflation predicted five recessions.

Table 5. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the lagged inflation rate time series. Source: Own calculations based on FRED and OECD data

Lag'd π	θ_0	θ_1	σ^2	P>|z| θ_0	P>|z| θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	4.42	-0.02	0.41	0.00	0.29	0.00	0.9	0.076
Reg.2	0.67	-0.00	20.0	0.494	0.967	0.00		

The Markov state probabilities D(t) at time t, where t = 1 quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

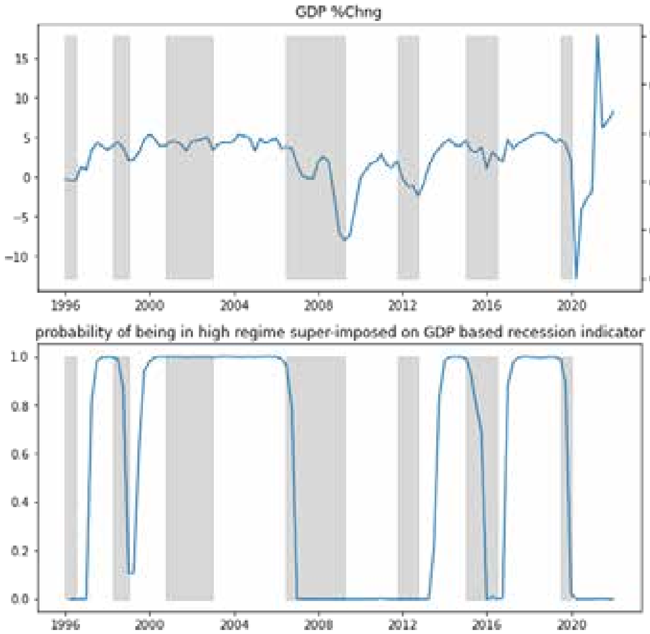

Figure 6. Smoothed probabilities of a recession, with lagged inflation rate as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regime 1 and 2 models of the GDP as y(t) and the Industrial production index m(t):
Regime 1: GDP %Change $= -0.44 + 0.286IPI + 1.159$;
Regime 2: GDP %Change $= 1.90 + 0.356IPI + 9.81$

From Table 6, the probability matrix is defined as: $P = \begin{bmatrix} 0.9359 & 0.0641 \\ 0.0421 & 0.9579 \end{bmatrix}$, the expected duration for regime 1 is 15.61 quarters, regime 2 is 23.73 quarters.

Table 6. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the Industrial Production Index (IPI) time series. Source: Own calculations based on FRED and OECD data

IPI	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	-0.44	0.286	1.159	0.309	0.00	0.00	0.936	0.042
Reg.2	1.90	0.356	9.81	0.00	0.00	0.00		

The 1-2 transition probability steady state is, 0.6035769, while the 1-1 transition probability is 0.396423given the 0.5 initial conditions of D(t).

Figure 7 illustrates the smoothed recession probabilities with the industrial production index being the explaining variable m(t) of the GDP %Change.

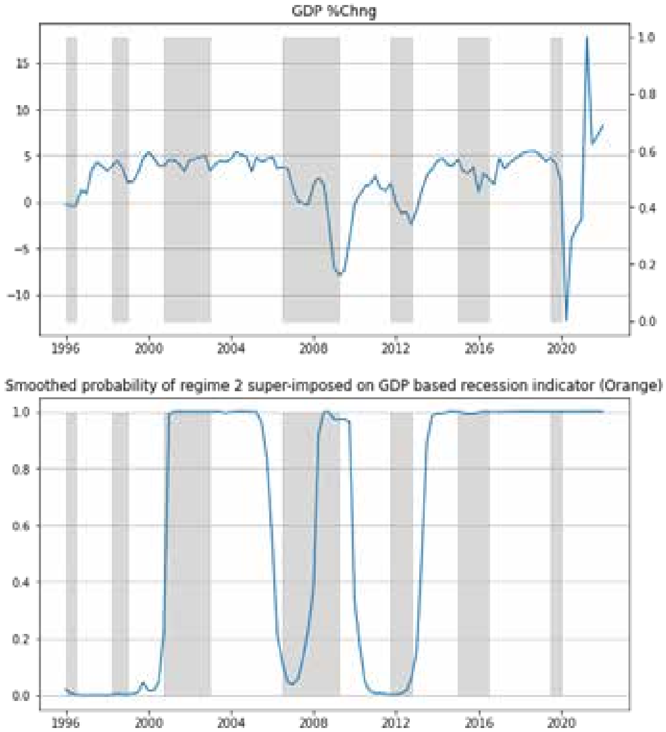

Figure 7. Smoothed probabilities of a recession, with Industrial production index (IPI) as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regime 1 and 2 models of the GDP as y(t) and the Lagged Industrial production index m(t):

Regime 1: GDP %Change $= -0.4657 + 0.2907\text{Lagged}IPI + 1.7076$;

Regime 2: GDP %Change $= 2.0451 + 0.0398\text{Lagged}IPI + 7.8434$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9659 & 0.0341 \\ 0.0302 & 0.9698 \end{bmatrix}$, the expected duration for regime 1 is 29.30 quarters, while the expected duration of regime 2 is 33.15 quarters (Table 7).

Table 7. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the Lagged Industrial Production Index (IPI) time series. Source: Own calculations based on FRED and OECD data

Lag'd IPI	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	-0.46	0.29	1.7	0.04	0.0	0.00	0.96	0.03
Reg.2	2.04	0.39	7.84	0.00	0.00	0.00		

The Markov state probabilities D(t) at time t, where t = 1 quarter, are given by Figure 8.

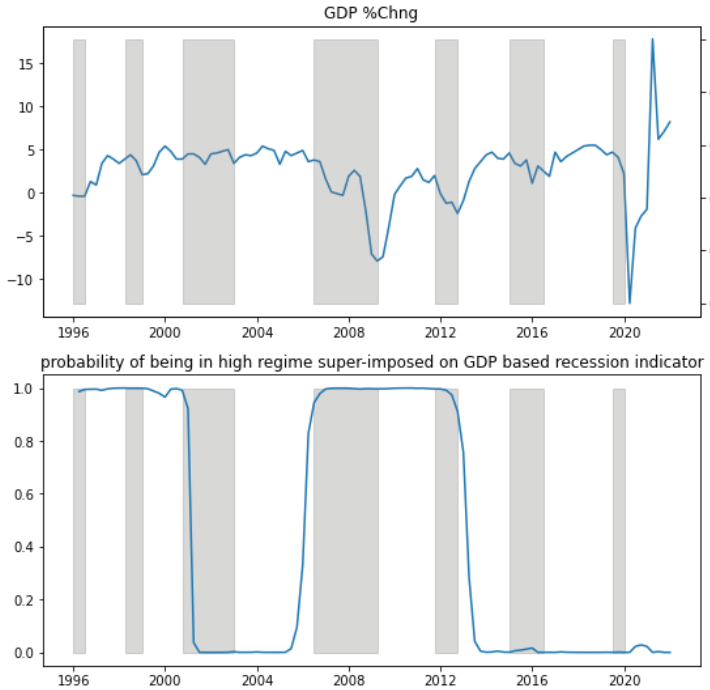

Figure 8. Smoothed probabilities of a recession, with lagged Industrial production index (IPI) as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regime 1 and 2 models of the GDP as y(t) and the BUX year-on-year change m(t):
Regime 1: GDP %Change $= 0.79 - 0.42BUXYoY + 1.99$;
Regime 2: GDP %Change $= 2.732 + 2.415BUXYoY + 13.74$

The probability matrix is defined as: $P = \begin{bmatrix} 0.8945 & 0.1055 \\ 0.0351 & 0.9649 \end{bmatrix}$, the expected duration for regime 1 is 9.48 quarters, while the expected duration of regime 2 is 28.51 quarters (Table 8).

Given the 50% chance of regimes 1 and 50% chance of regime 2, following 100 iterations, the state probability reaches the steady state: 0.2496 of probability of going to state 2 after state 1 and 0.750355 probability of remaining in state 1 after state 1.

Table 8. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the BUX year-on-year percentage change time series. Source: Own calculations based on FRED and OECD data

BUX YoY%	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg. 1	0.79	-0.42	1.99	0.097	0.55	0.07	0.894	0.035
Reg.2	2.732	2.415	13.744	0.00	0.10	0.00		

The Markov state probabilities D(t) at time t, where t = 1 quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse, Figure 9.

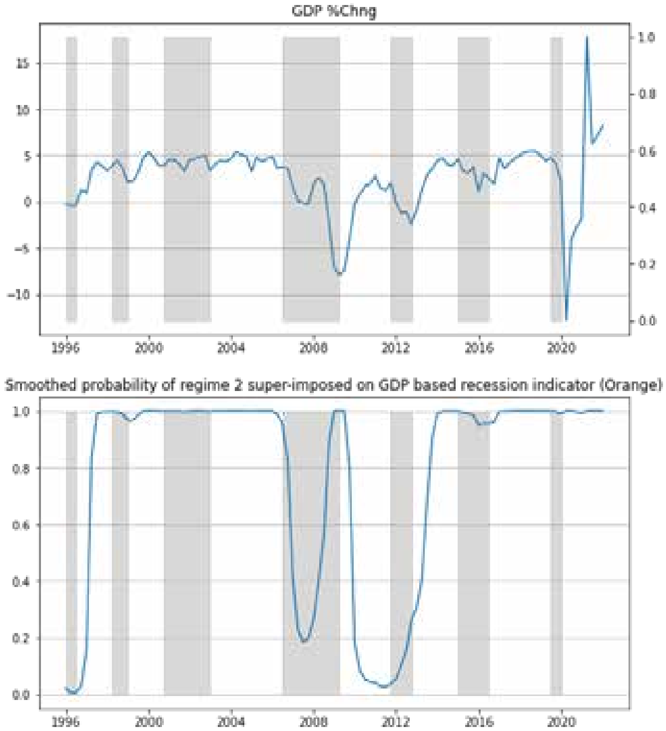

Figure 9. Smoothed probabilities of a recession, with BUX YoY% as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regime 1 and 2 models of the GDP as y(t) and the lagged BUX year-on-year change m(t):

Regime 1: GDP %Change $= 4.17 + 0.253\text{Lagged}BYoY + 0.539$;

Regime 2: GDP %Change $= 0.0014 + 2.769\text{Lagged}BYoY + 19.22$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9363 & 0.0637 \\ 0.0639 & 0.9361 \end{bmatrix}$, the expected duration for regime 1 is 15.70 quarters, while the expected duration of regime 2 is 15.65 quarters (Table 9).

Table 9. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the Lagged BUX year-on-year percentage change time series. Source: Own calculations based on FRED and OECD data

Lag-d BUX YoY	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	4.17	0.253	0.539	0.00	0.45	0.00	0.936	0.063
Reg.2	0.0014	2.769	19.22	0.99	0.45	0.00		

The Markov state probabilities D(t) at time t, where t = 1 quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse, Figure 10a

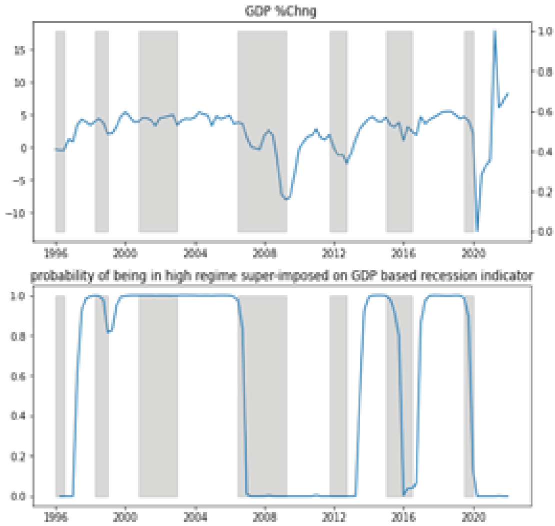

Figure 10. Smoothed probabilities of a recession, with Lagged BUX YoY% as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Regime 1: GDP %Change $= 4.37 + 0.00241 YSpread + 0.40$;
Regime 2: GDP %Change $= 0.6287 + 0.07 YSpread + 21.80$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9294 & 0.0716 \\ 0.0616 & 0.9384 \end{bmatrix}$, the expected duration for regime 1 is 14.16 quarters, while the expected duration of regime 2 is 16.22 quarters (Table 10).

Table 10. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the 10Year and 3Year yield spread time series. Source: Own calculations based on FRED and OECD data

YSpread	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Reg.1	4.37	0.00	0.40	0.00	0.97	0.00	0.92	0.061
Reg.2	0.62	0.07	21.80	0.38	0.89	0.00		

Given the 50% chance of regimes 1 and 50% chance of regime 2, following 100 iterations, the state probability reaches the steady state: 0.4683258 of probability of going to state 2 after state 1 and 0.531674719 probability of remaining in state 1 after state 1.

The Markov state probabilities D(t) at time t, where t = 1 quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse, Figure 11.

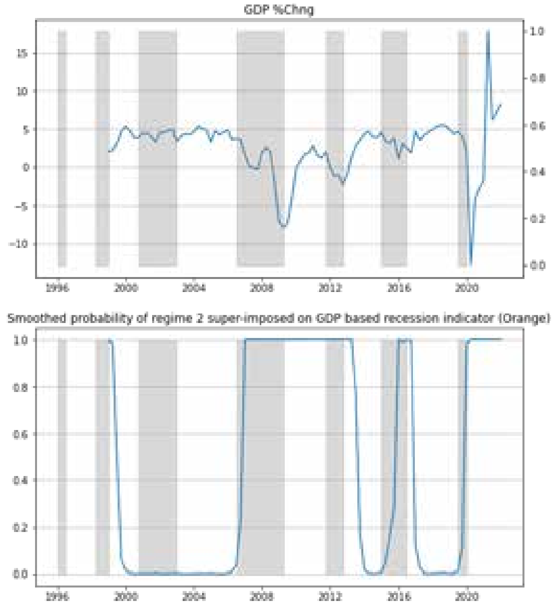

Figure 11. Smoothed probabilities of a recession, with YSpread as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculation

Regime 1 and 2 models of the GDP as y(t) and the Lagged Yspread yields m(t):

Regime 1: GDP %Change $=\ 4.33 + 0.0638\text{Lagged}YSpread + 0.48$;

Regime 2: GDP %Change $=\ -0.3363 + 1.9894\text{LLagged}YSpread + 19.41$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9481 & 0.0529 \\ 0.0614 & 0.9386 \end{bmatrix}$, the expected duration for regime 1 is 19.27 quarters, while the expected duration of regime 2 is 16.28 quarters (Table 11).

Table 11. Estimation of the Markov regime switching model. y(t) is the GDP variable, while m(t) is the Lagged 10Year and 3Year yield spread time series. Source: Own calculations based on FRED and OECD data

Lag'dYSpread	θ_0	θ_1	σ^2	P>|z|θ_0	P>|z|θ_1	P>|z| σ^2	p_{11}	p_{21}
Regime 1	4.33	0.06	0.48	0.00	0.30	0.00	0.948	0.061
Regime 2	-0.3	1.98	19.41	0.66	0.01	0.00		

The Markov state probabilities D(t) at time t, where t = 1 quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse, Figure 12.

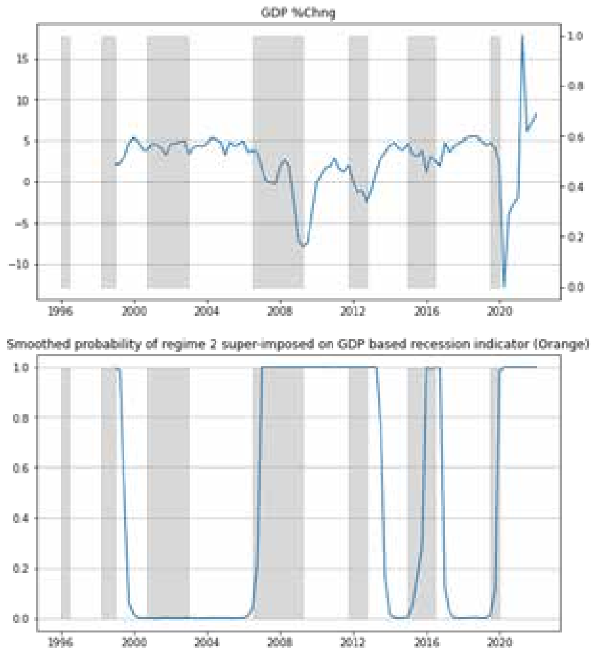

Figure 12. Smoothed probabilities of a recession, with Lagged YSpread as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculation

6. Discussion

The parameter estimates of the linear MSDR model are presented in Tables 3-11. The selection of the exogenous variables is justified in section 4 of the research. The estimation period for the time series is 105 quarters or 26 years and 3 months. Figures 3-12 plot the smoothed recession probabilities compared to actual recession represented by grey bars on the plot. Let us examine the results obtained for each of the exogenous variables - m(t):

Unemployment:

Firstly, the θ_1 value for regime 1 is negative, this shows that during an expansion the unemployment rate tends to drop. This follows the conventional laws of Keynesian economics, whereby, because of the wide availability of credit, companies can increase the labor force, decreasing the unemployment rate. However, θ_1 for regime 2 is also negative, moreover, its slope is steeper than that of regime 1. Following the previous reasoning, this means that during a recession unemployment decreases at a faster rate than during an expansion. Secondly, the variance of the error term is significantly higher during a recession – this points to stronger propagation of the shocks. The graph of the smoothed recession probabilities captures 5 out of the 7 recessionary periods of the economy – a 71% predictive power, which is by far the best result out of the panel data. The fallbacks of the smoothed probability results are that the graph doesn't capture the early 2000's recession and the 1999 recession, and for some reason, the regime remains at state 2 up till now. Could this mean that since the COVID-19 crisis the dynamics of the Unemployment rate remained in regime 1?

Inflation:

We start with the regression coefficients for regime 2: we obtain θ_1 and assert that it is positive. Indeed, this matches with theory – higher inflation leads to poor credit conditions in an economy, resulting in a decrease in the output rate. In the expansionary regime 1, θ_1 has a negative slope, indicating that inflation steadily decreases. Alternatively, when the inflation rate is lagged back 1 quarter, both regimes feature a negative θ_1 variable. The variance of regime 2 is high compared to regime 1. At the same time, rising inflation is not to be associated with the certainty of a recessionary state. For instance, a radical example would be that according to modern monetary theory, if the overall growth rate of the economy remains higher than that of inflation, it is possible to sustain the model indefinitely – attaining an endless period of expansion. It is important to note that the measure of expansion (regime 1) is not a low, zero, or even, negative inflation rate. Deflation is just as harmful, if not even more harmful in an economy as inflation. The inflation variable captures 6 of the 7 recessionary periods in Hungary, which at first glance, may be an even better result than a model with the unemployment rate as the exogenous variable. However, examining the plot in detail, we can see that the recession probability remained high throughout the 2008-2012 business cycle encompassing two recessions and one expansionary period. The model, therefore, failed to predict the expansion between 2009 and 2012. We conclude that the inflation model is on par with the unemployment model in the overall signal to noise ratio.

Industrial Production Index:

The estimates provided by the MSDR training results summary indicate that notwithstanding the state of the business cycle, whether the exogenous variable is lagged or not, the slope of the regime equations remains positive. This leads to another questionable conclusion that according to our model, while the negative GDP growth slows down the growth of the IPI, it does not make it negative. The IPI showed the recession between 2001 and 2003, corresponding to the GDP decline, it also switched to the high regime in the second half of the 2008 recession, missed the 2012 European credit crisis, and, if such an interpretation can be made, predicted the 2016 GDP decline at least 6 quarters ahead. From that moment, it remained in state 2 – missing the expansionary periods between 2016 and 2020. The lagged IPI performed even worse, with only capturing 4 of the recessions along the examined time horizon.

BUX YoY% change

The regime 1 coefficient for the BUX growth rate exogenous variable is -0.42 with a statistical significance of 0.55, which is far more than the 5% threshold indicating statistical insignificance in favor of the null hypothesis. For the lagged BUX YoY variable, the p-value is also not statistically significant. According to the results in Table 8, the duration of regime 2 is 3 times longer on average than of regime 1. The smoothed recession probability graph indicates that the model captures 5 of the 7 recessions in the examined timeframe, but at the same time misses 3 expansionary periods. It assumed in this research that the stock market is a strong leading indicator. One explanation as to why regime 2 occurs so often is related to the effect of volatility. The problem could be with the data: if the quarterly period ending values of the BUX were represented in a weekly or daily frequency, it would have been possible to smooth the data, eliminate short-term volatility and filter for a quarterly frequency. In this case, however, it is complicated to say what part of the quarterly BUX value is 'noise' and what part of it is actual trend. Therefore, as a possibility of further extending this work, following the data manipulation procedure mentioned before, the test should be repeated and compared with the results obtained in this research.

10-Year and 3-Year Hungarian sovereign bond yield spreads.

The MSDR training model produced one of the most interesting outputs for the yield spread exogenous variable. To reiterate, yield curve inversions detected from spreads of long and short maturity bonds have a capability of predicting recessions. Therefore, the theoretical specifications of regimes 1 and 2 couldn't have been clearer: regime 1 for positive spreads and regime 2 for negative spreads. The θ_1 value for the yield spread time series in both regimes 1 and 2 was extremely close to 0. This can be interpreted as: yield spreads have a negligible influence on GDP growth. When the economy is in an expansion, the yield spread is very small, when the economy is in a recession, the spread grows, but according to our model, only slightly. The probability of the persistence of a low regime is nearly 95%, while the persistence of a high regime is 94%. The smoothed recession probabilities capture 5 of the 6 recessions but miss 2 expansionary periods. As a possible extension to the dataset and the approach, it might be worth to do the same procedure with the 15 year and 3 month spreads, the 15 year and 3 year spreads, and the 10 year and 3 month spreads. By expanding the maturities of the sovereign bonds, it may be possible to better ascertain how investor sentiment correlates with Hungarian business cycles.

From the methodological standpoint, there are a few remarks to be made:

We set the initial D(t) probability to 0.5 on both regimes, indicating that our model has an equal chance of being in a recession and an expansion at time t_0. We follow this approach only for purposes of obtaining the state probability distribution. It can be argued, however, that the initial probability on the regimes is not 50%. For example, we can look back at recessionary periods for a given country's economy and count the average duration of a recession against the average duration of an expansion and adjust the initial probabilities by that value. In this aspect this research needs further extensions.

Homoscedasticity has a normal distribution – this is a questionable assumption within the model. What is the concrete factor in the economy m(t) that is referred to as the 'hidden variable' – what is the economic significance of the regime switch particularly for the Hungarian economy?

A key characteristic of the applied Markov regime switching dynamic factor model is the assumption that within the transition matrix the transition probabilities are assumed to be constant. Filardo (1994) addresses this issue by allowing the probabilities to vary over time. For example, p_{22} in time t does not equal p_{22} in $t + n$ steps, where n is the time-step. Exploring regime switching within the time varying transition probability (TVTP) framework can yield some interesting results for analyzing the regime switches of yield curves and GDP or GNP time series.

7. Conclusions

This paper reevaluates the idea that the state of the business cycle can be determined based on the co-movements of macroeconomic indicators. By applying the

Markov regime-switching dynamic model, it is possible to isolate the hidden variable and the one which is directly observed by the econometrician. The hidden variable is modeled through a two-state Markov chain, while the observed variable is modeled through a regression. We build a linear regression model with a mean, intercept and error term, where the mean is replaced by the product of some explanatory variable and a coefficient. The transition matrix for the time series is estimated through the MLE and the smoothed recession probabilities are graphed against actual recession periods given by the OECD for visual inspection and conclusions. The Empirical estimation shows that the panel data have different statistical parameters depending on the growth or recessionary state of the system, which in our case is the Hungarian economy. The hidden part of the Markov switching dynamic regression model explained most of the variance in the GDP % change, that a simple regression with panel data variables wouldn't be able to. We have determined the expected durations of the recessions by plugging in the panel data variables as the exogenous term of our model. The GDP corresponds to the business cycles the most. when the exogenous variable is the unemployment rate. To determine possible long-term correlations of the GDP with the business cycle, the exogenous variables were lagged by 1 quarter. Generally, this extension did not result in a higher predictive capability, than by not lagging the variable.

The further development of the Markov chain method is important because this allows us to analyze the unobservable variable in greater detail. The lagged maximum likelihood function of the 10 and 3 year spreads indicates that the Hungarian economy is in a recession, which at the time of writing this paper it is in a recession. However, the unemployment rate has been determined to be the most accurate indicator, which is accurate for 8 out of the 13 cycles. From a legislative standpoint, it is worth to integrate a constantly updating maximum likelihood function that would switch if a recessionary or expansionary period is being detected. Also, it Is worth considering more frequent time series. Within the scope of this research, we have worked with quarterly time series, if for instance, we were to take the jobless claims instead of the quarterly unemployment rate, which is a monthly indicator and eliminate the noise and smooth it, we believe that it would be a much better 'live' version of whether the economy is bound for a recession. Further methodological developments in the case of Hungarian business cycle estimation could be in applying Filardo's TVTP framework for the panel data.

Appendix A: Additional figures for the MSDR model with Unemployment as exogenous variable.

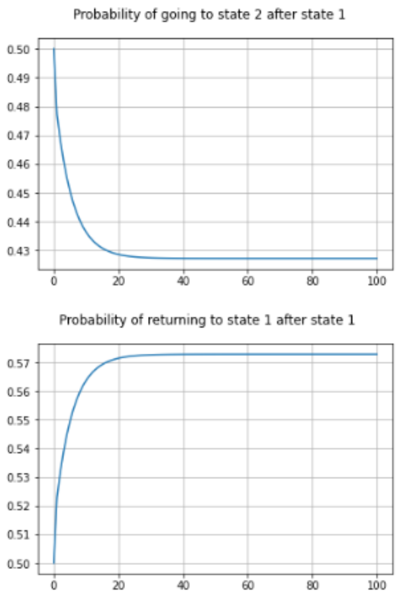

Figure 13. State probability distribution for GDP %Change & Unemployment time series

Appendix B: Additional figures for the MSDR model with Inflation rate as exogenous variable.

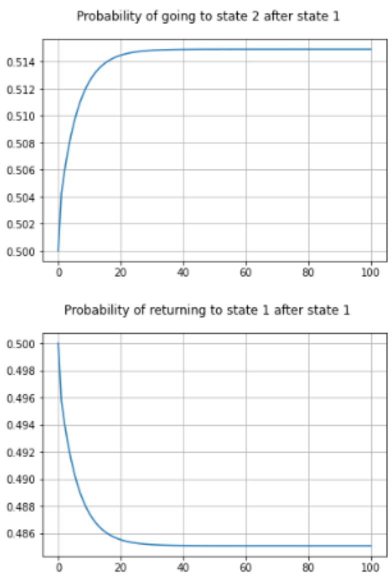

Figure 14. State probability distribution for GDP %Change & lagged Unemployment time series

Appendix C: Additional figures for the MSDR model with IPI as exogenous variable.

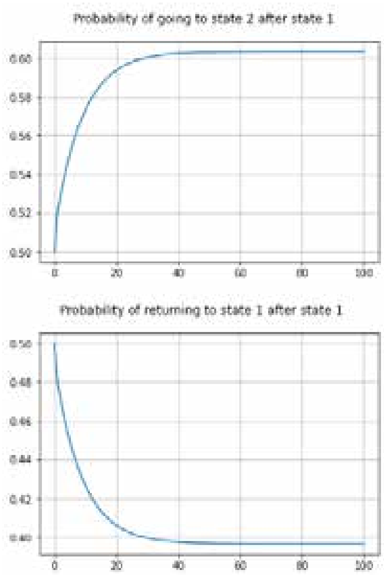

Figure 15. State probability distribution for GDP %Change & IPI time series

Appendix D: Additional figures for the MSDR model with BUX as exogenous variable.

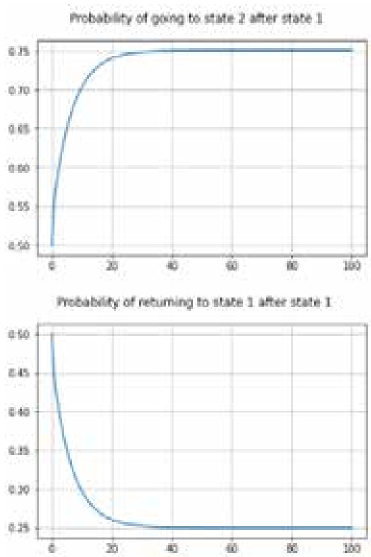

Figure 16. State probability distribution for GDP %Change & BUX YoY change time series

Appendix E: Additional figures for the MSDR model with 10Year and 3Year sovereign bond yield spread as exogenous variable.

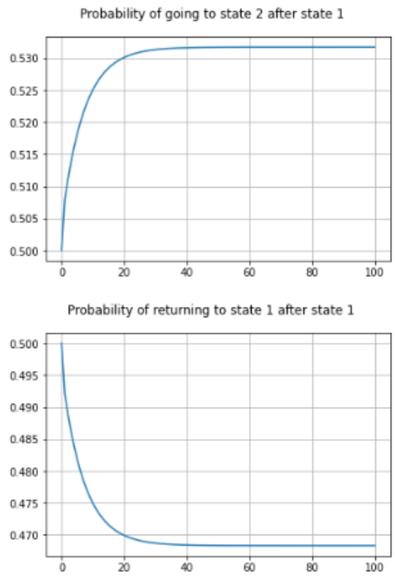

Figure 17. State probability distribution for GDP %Change & BUX YoY change time series

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